AN INVESTIGATION OF THE EQUATIONS OF MOTION METHOD FOR THE EVALUATION OF THE LONGITUDINAL DYNAMIC STABILITY DERIVATIVES OF THE F-80A FROM TRANSIENT RESPONSE DATA

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Report No. 207

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ACKNOWLEDGMENT

The authors extend their appreciation to Professor E. Seckel for his advice and guidance and to the Cornell Aeronautical Laboratory, Inc., for making the F-80A transient response data available for this investigation.

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SUMMARY

Transient response data obtained from the flight tests of an F-80A airplane were analyzed for the longitudinal stability derivatives by the Equations of Motion Method in order to determine whether or not the method is practicable. Other considerations include the possibility of determining the moment of inertia of the aircraft from transient response data and a relative evaluation of the type of input that should be used to obtain the transient data.

All data considered were obtained at a constant center of gravity position and essentially the same altitude and Mach Number.

The method is considered to be a practical means of obtaining the longitudinal stability derivatives of an airplane provided certain precautions are observed. The accuracy of the moment of inertia determination is somewhat less than that obtainable from ground tests. In general the step function data yielded better results than the impulse data.

The values of the stability derivatives and the inertia parameter, h, are as follows:

	Step Input	Impulse Input
C_{L_α}, per radian	6.22	5.91
C_{m_δ}, per radian	-0.96	-0.86
C_{m_α}, per radian	-0.324	-0.357
$C_{m_{d\dot{\alpha}}}$, per radian	-0.023	-0.033
$C_{m_{d\theta}}$, per radian	-0.044	-0.027
h	+0.015	+0.010

INTRODUCTION

In recent years a great deal of attention has been directed toward determining the stability derivatives of missiles and airplanes. Steady-state testing techniques and wind tunnel information are used effectively to determine the stability characteristics for subsonic aircraft. However, with the coming of jet and rocket powered aircraft and with the resulting transonic and supersonic speeds, it has become increasingly difficult to obtain reliable information from wind tunnel tests. Also the inherent difficulty in achieving steady flight conditions in the transonic and supersonic speed ranges, combined with the relatively brief testing period available, severely limits the use of steady-state testing procedures. It is, therefore, highly desirable that some satisfactory means of analysis of transient response data be developed.

Present day methods of analyzing transient data can be broadly classified as follows:

(1) Methods dependent upon the solution of the equations of motion of the airplane

(2) Curve fitting methods

Outlines of the variations in each of the above classifications may be found in Ref. 1 and 2.

The primary objective of the present investigation is to determine the feasibility of analyzing transient response data

by the "Equations of Motion Method" in order to determine the longitudinal stability derivatives of an airplane. Secondary considerations include (1) the relative merit of the response to an impulse input as compared to that of a step input and (2) the possibility of determining the moment of inertia of the aircraft from the equations of motion.

Transient response data for an F-80A airplane were obtained from Cornell Aeronautical Laboratory, Inc., (Fig. 1 and 2). The flight testing was performed by Cornell Aeronautical Laboratory, Inc., and information pertaining to the instrumentation and method of tests can be obtained in Ref. 3.

The present report is limited to an analysis of two sets of data consisting of the response due to an impulse input to the elevator and the response due to a step deflection of the elevator. The stability derivatives obtained are compared to theoretical values and those previously determined from steady state oscillation data by Cornell Aeronautical Laboratory, Inc., (Ref. 3)

The Mach Number, altitude, and cg position are essentially the same for each run.

CONVENTIONS

AXES

The wind axes or stability axes are used. The origin O lies at the center of gravity of the airplane. The OX and OZ axes lie in the plane of symmetry and OY is perpendicular to it. The OX axis always points in the direction of motion or into the relative wind.

X axis, or longitudinal axis, positive forward

Y axis, or transverse axis, positive along the right wing

Z axis, or normal axis, positive downward

LINEAR DISPLACEMENTS

A linear displacement along a positive reference axis is considered to be positive.

ANGULAR DISPLACEMENTS AND MOMENTS

An angular displacement or moment which is clockwise when viewed from the origin looking along a positive reference axis is considered to be positive.

VELOCITIES AND ACCELERATIONS

Velocities and accelerations, either linear or angular, are considered positive in the same sense as the corresponding displacements.

CONTROL SURFACE DEFLECTIONS

Positive elevator angle is associated with a downward movement of the elevator trailing edge.

TAIL LOAD

Positive tail load is associated with a downward load on the horizontal tail.

NOTATION

The symbols used in this report are essentially those employed by the National Advisory Committee for Aeronautics. Symbols pertinent to this report are listed below for convenience:

V_o	true airspeed, ft. per sec.
q	pitching velocity, radians per sec.
I_y	moment of inertia about Y axis, slug ft.^{2}
K_y	radius of gyration about Y axis, ft.
m	mass of airplane, slugs
W	weight of airplane, lbs.
c	mean aerodynamic chord, ft.
g	acceleration of gravity, ft. per sec.^{2}
S	wing area, ft.^{2}
t	time, sec.
n_z	incremental normal acceleration, g's
α	wing angle of attack, radians
δ	elevator deflection, radians

ϵ	downwash angle, radians
θ	angle of pitch, between X axis and the horizontal, radians
ρ	mass density of air, slugs per ft.^{3}
μ	airplane density factor, $\mu = m / \rho S c$
τ	aerodynamic time unit, sec., $\tau = m / \rho S V_o$
d	differential operator, $d = d / d\frac{t}{\tau} = \tau (d / dt)$
l_t	tail length, ft. (c.g. to 25% tail chord)
C_L	initial airplane lift coefficient, $C_L = W / \frac{1}{2} \rho S V_o^2$
C_m	airplane moment coefficient, $C_m = M / \frac{1}{2} \rho S V_o^2 c$
h	moment of inertia parameter, $h = 2K_y^2 / \mu c^2$
C_{L_α}	slope of the lift curve, $C_{L_\alpha} = dC_L / d\alpha$
C_{m_α}	static longitudinal stability criterion, $C_{m_\alpha} = dC_m / d\alpha$
$C_{m_{d\alpha}}$	rate of change of pitching moment coefficient with rate of change of angle of attack with respect to t/τ
$C_{m_{d\theta}}$	airplane damping in pitch
C_{m_δ}	elevator control power, $C_{m_\delta} = \partial C_m / \partial \delta$
$d\epsilon/d\alpha$	rate of change of downwash with respect to α
i_t	angle of incidence of the tail, radians
$C_{m_{i_t}}$	rate of change of moment coefficient with respect to i_t
N_t	observed tail load, lbs.
l	moment arm of observed tail load, ft. (c.g. to 50% tail chord)
W_t	weight of tail outboard of strain gage fittings
C_{N_t}	tail load coefficient, $C_{N_t} = N_t / \frac{1}{2} \rho V_o^2 S$

$$\left.\begin{array}{l} C_{L_{\alpha_1}} \\ C_{m_{\alpha_1}} \\ C_{m_{d\theta_1}} \\ C_{m_{d\alpha_1}} \end{array}\right\}$$ Subscript, 1, denotes that the stability derivatives are for the "tail-off" airplane.

THEORY AND ANALYSIS

Most of the applications of mathematical analysis to the motion of an airplane have been for the purpose of determining the airplane's response to a disturbance. This study considers the inverse of the above problem, that is, the determination of the stability derivatives of the airplane from the longitudinal equations of motion when response information is given.

In setting up the longitudinal equations of motion the following assumptions were made:

(1) The airplane remains at constant airspeed.

(2) The equations are linear with constant coefficients.

(3) Measurements of the forcing function and responses are subject to random errors only.

Because of the first assumption, the longitudinal equations of motion for the short period transient response may be written in terms of the lift and moment equations only.

$$(1) \quad \frac{C_{L\alpha}}{2}\alpha + d\alpha - d\theta - \frac{C_{m_\delta}\delta}{2\,l_t/c} = 0$$

$$(2) \quad C_{m_\alpha}\alpha + C_{m_{d\alpha}}d\alpha + C_{m_{d\theta}}d\theta - hd^2\theta = -C_{m_\delta}\delta$$

Since incremental normal acceleration, n_z, is more readily determined in flight than angle of attack, α, the following relation can be utilized to incorporate n_z in place of α.

$$(3) \quad \alpha = -\frac{1}{C_{L\alpha}}\left[C_L n_z - \frac{C_{m_\delta}}{l_t/c}\right]$$

Also, since the pitching velocity is usually measured by the rate gyro in terms of $q = d\theta/dt$, the following relation may be used for $d\theta$.

$$(4) \quad d\theta = \tau q$$

Substituting Equations (3) and (4) into Equation (1), the lift equation becomes

$$(5) \quad C_L \frac{C_{L\alpha}}{2} n_z + C_L\, dn_z + \tau C_{L\alpha} q - \frac{C_{m\delta}}{l_t/c} d\delta = 0$$

With n_z, dn_z, q, and $d\delta$ available from the transient response data, $C_{L\alpha}$ can now be determined. $C_{m\delta}$ can also be determined from the above equation, but its accuracy will be limited by the fact that it appears in a constant coefficient that is quite a small number.

For convenience, Equation (5) is regrouped in the following form:

$$(6) \quad \frac{C_{L\alpha}}{C_L} v + dn_z - \frac{C_{m\delta}}{l_t/c} \quad \frac{1}{C_L} d\delta = 0$$

where $\quad (6a) \quad v = \frac{C_L}{2} n_z + \tau q$

Because of the relatively poor rate gyro measurement of q on the oscillograph as shown in Fig. 1 and 2, the use of q in the solution of the moment derivatives is avoided. By solving for $d\theta$ in Equation (1) and substituting $d\theta$ and also α from

Equation (3) into Equation (2) the moment equation becomes

$$(7) \quad \left(C_{m\alpha} + \frac{C_{L\alpha}}{2} C_{m_{d\theta}}\right) n_z + \left(C_{m_{d\alpha}} + C_{m_{d\theta}} - h \frac{C_{L\alpha}}{2}\right) dn_z$$

$$- hd^2 n_z - \frac{C_{m\delta}}{l_t/c} \frac{1}{C_L} \left(C_{m\alpha} + \frac{l_t}{c} C_{L\alpha}\right) \delta$$

$$- \frac{C_{m\delta}}{l_t/c} \frac{1}{C_L} \left(C_{m_{d\alpha}} + C_{m_{d\theta}}\right) d\delta + \left(\frac{C_{m\delta}}{l_t/c} \frac{h}{C_L}\right) d^2\delta = 0$$

In order to avoid the inherent inaccuracy of determining second derivatives from the transient response data, Equation (7) is integrated once. The resulting moment equation is as follows:

$$(8) \quad \left(C_{m\alpha} + \frac{C_L}{2} C_{m_{d\theta}}\right) \int n_z d\, t/_\tau + \left(C_{m_{d\alpha}} + C_{m_{d\theta}} - h \frac{C_{L\alpha}}{2}\right) n_z$$

$$- hdn_z - \frac{C_{m\delta}}{l_t/c} \frac{1}{C_L} \left(C_{m\alpha} + \frac{l_t}{c} C_{L\alpha}\right) \int \delta \, d\, t/_\tau$$

$$- \frac{C_{m\delta}}{l_t/c} \frac{1}{C_L} \left(C_{m_{d\alpha}} + C_{m_{d\theta}}\right) \delta + \left(\frac{C_{m\delta}}{l_t/c} \frac{h}{C_L}\right) d\delta = 0$$

With the inertia parameter, h, and all the response data known, this equation can be solved for the "lumped constants" or combinations of moment derivatives.

In order to determine uniquely the values of the moment derivatives, the additional equations required are found by use of the observed tail load, as suggested by Ref. 4. "Suppose that the horizontal tail of the airplane were totally ineffective in producing any aerodynamic forces or moments on the airplane.

Suppose then that the airplane in this condition were subjected somehow to loads at the tail exactly equal to the tail loads observed in any practical maneuver. It is evident that the response would be exactly the observed one." The longitudinal equations of motion for the airplane in this condition are

$$(9) \quad \frac{C_{L\alpha_1}}{2} \alpha + d\alpha - d\theta - \frac{C_{n_t}}{2} = 0$$

$$(10) \quad C_{m_{\alpha_1}} + C_{m_{d\alpha_1}} d\alpha + C_{m_{d\theta_1}} d\theta - hd^2\theta = -\frac{1}{c} C_{n_t}$$

The equation of α in terms of n_z now becomes

$$(11) \quad \alpha = \frac{C_L}{C_{L\alpha_1}} \left[\frac{C_{n_t}}{C_L} - n_z \right]$$

Substituting Equations (4) and (11) into Equation (9), the "tail-off" lift equation becomes

$$(12) \quad \frac{C_L}{C_{L\alpha_1}} dy = \tau q + \tfrac{1}{2} C_{n_t} - \frac{C_L}{2} y$$

where $$(12a) \quad y = \frac{1}{C_L} C_{n_t} - n_z$$

With all variables known from the transient response data, $C_{L\alpha_1}$, may be uniquely determined.

By solving for $d\theta$ in Equation (9) and substituting $d\theta$ and

also α from Equation (11) into Equation (10) the "tail-off" moment equation becomes

$$(13)\quad \left(C_{m\alpha_1} + \frac{C_{L\alpha_1}}{2} C_{md\theta_1}\right) y$$

$$+ \left(C_{md\alpha_1} + C_{md\theta_1} - h \frac{C_{L\alpha_1}}{2}\right) dy$$

$$- hd^2y - \left(\frac{C_{md\theta_1}}{2} \frac{C_{L\alpha_1}}{C_L}\right) C_{n_t}$$

$$+ \left(\frac{h}{4} C_{L\alpha_1}\right) dC_{n_t} + \left(\frac{C_{L\alpha_1}}{C_L} \frac{1}{c}\right) C_{n_t} = 0$$

Since the derivative $C_{md\alpha_1}$ is small enough to be considered negligible, it is omitted hereafter.

Once again, to avoid determining second derivatives from the transient response data, Equation (13) is integrated. The resulting equation is as follows:

$$(14)\quad \left(C_{m\alpha_1} + \frac{C_{L\alpha_1}}{2} C_{md\theta_1}\right) \int y d\, t/\tau$$

$$+ \left(C_{md\theta_1} - h \frac{C_{L\alpha_1}}{2}\right) y - hw =$$

$$\frac{C_{L\alpha_1}}{C_L} \left(\frac{C_{md\theta_1}}{2} - \frac{1}{c}\right) \int C_{n_t} d\, t/\tau$$

where $w = (dy - \frac{C_{L\alpha_1}}{2C_L} C_{n_t})$ is employed for simplification of calculations.

In the last coefficient of Equation (14) the term $\frac{C_{m_{d\theta_1}}}{2}$ is small compared to 1/c and is omitted. This allows for the solution of h and hence the moment of inertia, I_y, from the flight test data.

With all the response data variables known, Equation (14) can be solved for h, $C_{m_{\alpha_1}}$, and $C_{m_{d\theta_1}}$.

It is now possible to set down six equations from which the unique moment derivatives may be found.

$$(15) \quad C_{m_\alpha} = C_{m_{\alpha_1}} + C_{m_{i_t}} \left[1 - \frac{d\varepsilon}{d\alpha} \right]$$

$$(16) \quad C_{m_{d\theta}} = C_{m_{d\theta_1}} + \frac{1}{\mu} \frac{l_t}{c} C_{m_{i_t}}$$

$$(17) \quad C_{m_{d\alpha}} = \frac{1}{\mu} \frac{l_t}{c} C_{m_{i_t}} \frac{d\varepsilon}{d\alpha}$$

$$(18) \quad C_{m_{d\alpha}} + C_{m_{d\theta}} - h \frac{C_{L\alpha}}{2} = \text{Constant, coefficient}$$

from Equation (8)

$$(19) \quad C_{m_\alpha} + \frac{C_{L\alpha}}{2} C_{m_{d\theta}} = \text{Constant, coefficient from}$$

Equation (8)

$$(20) \quad \frac{C_{m_\delta}}{l_t/c} \quad \frac{1}{C_L} \quad (C_{m_\alpha} + \frac{l_t}{c} \quad C_{L_\alpha}) = \text{Constant, coef-}$$

ficient from Equation (8)

These six simultaneous equations can be solved for the six unknowns, C_{m_α}, $C_{m_{d\alpha}}$, $C_{m_{d\theta}}$, C_{m_δ}, $C_{m_{i_t}}$, and $d\varepsilon/d\alpha$.

The solutions of the equations to determine C_{L_α}, h, the "tail-off" coefficients, and the combined moment derivatives all depend upon the use of redundant data. The method of least squares is employed as the procedure in handling the redundant data. The authors are well aware that the method of least squares is not applied in a rigorous fashion. Although all errors in the data were considered random, it is quite possible that systematic errors are present.

In some instances variables are combined to form new variables (Equations 6a, 12a). Also the errors are assumed to be in one variable only. (See Appendix C.) However, it is believed that application of least squares as used throughout the analysis is a reasonable way of determining the stability derivatives from the redundant data.

RESULTS AND DISCUSSION

The results of this investigation are presented in the tables below. Results of previous investigations from both theoretical analysis and flight test data are also presented for comparison.

	*Analytical Results	*Previous Flight Test Results	Step Function Results (Run 5301)	**Impulse Function Results (Run 5068)
#C_{L_α}	6.43	6.19	6.22	5.91
$C_{L_{\alpha_1}}$	6.12	5.8	5.93	6.37
h	0.0133	—	0.0150	0.0100
$C_{m_{d\theta_1}}$	0	wide scatter -0.0124 to +0.248	- 0.0102	0.0176
$C_{m_{\alpha_1}}$	0.55	0.48	0.435	0.397
$C_{m_\alpha} + \frac{C_{L_\alpha}}{2} C_{m_{d\theta}}$	—	- 0.403	- 0.439	- 0.461
$C_{m_{d\alpha}} + C_{m_{d\theta}} - h\frac{C_{L_\alpha}}{2}$	—	- 0.090	- 0.1011	- 0.108
$\frac{C_{m_\delta}}{l_t/c}\ \frac{1}{C_L}\left(C_{m_\alpha} + C_{L_\alpha}\frac{l_t}{c}\right)$	—	-40.0	-41.16	-44.25
C_{m_δ}	- 0.63	- 0.86	- 0.96	- 0.86
C_{m_α}	- 0.29	- 0.35	- 0.324	- 0.357
$C_{m_{d\alpha}}$	- 0.012	- 0.022	- 0.023	- 0.0335
$C_{m_{d\theta}}$	- 0.024	- 0.032	- 0.044	- 0.0268
$C_{m_{i_t}}$	- 1.74	- 2.44	- 2.44	- 3.08
$d\varepsilon/d\alpha$	+ 0.51	+ 0.68	+ 0.683	+ 0.756

c.g. = 27% MAC   M = 0.707

* Approximate averages obtained from Ref. 3.

**The lift due to the deflection of the elevator was omitted in determining moment derivatives.

\# Derivatives are per radian.

As noted from the above table, the results of this investigation, for both the step function and the impulse function, are numerically comparable to those obtained previously. From these results it appears that the method used to analyze transient response data is practical and, with certain precautionary measures to be discussed throughout this section, the derivatives obtained are comparable in accuracy to those determined from frequency response methods.

The oscillograph records of the transient response data are shown in Fig. 1 and 2. All values of the basic variables therein were taken within a one-second interval after the elevator deflection was applied. The one-second interval was divided into twenty equally spaced points at which the readings of the variables were taken. The readings of pitching velocity and tail load were corrected for calibration error and the effect of tail weight in Appendices A and B, respectively.

The rate gyro measurement of q was exceptionally poor. The oscillatory q response was faired as shown in Fig. 1 and 2. For this reason the q data was used only where necessary, that is, in the calculation of C_{L_α} and $C_{L_{\alpha_1}}$.

<u>Response to the Step Deflection of the Elevator</u>

For the calculation of C_{L_α} , $C_{L_{\alpha_1}}$, h, and the "tail-off" derivatives, data at Points 3 through 20 were used in the least squares solution. The values of $C_{L_\alpha} = 6.22$ and $C_{L_{\alpha_1}} = 5.93$ agree quite well with the values $C_{L_\alpha} = 6.19$ and $C_{L_{\alpha_1}} = 5.8$

obtained previously. However, the inertia parameter, h = 0.0150, is approximately 12 per cent higher than h = 0.0133 as calculated from the previously known value of moment of inertia. There is no apparent reason for the discrepancy, and from a theoretical viewpoint it should be possible to obtain sufficiently accurate values of the inertia parameter.

Appendix D contains a more simple equation from which h can be determined. This equation could not be used to advantage in the present analysis because of its dependency on accurate rate gyro measurements of pitching velocity.

The combined moment derivatives were obtained by the solution of Equation (8) of the Theory and Analysis section of this report. Values of the basic variables were taken at the even numbered points in Fig. 1. The number of points used in the least squares solution was reduced to ten in order to reduce the amount of calculations involved.

An inspection of Fig. 1 indicates that the slope of the elevator response is practically negligible subsequent to Point 2. A first attempt at reading the slope after Point 2 gave a maximum and minimum $d\delta = \pm 0.000221$ per radian. When these very small values of $d\delta$ were used, completely absurd results for the combined moment derivatives were obtained. However, when the value of $d\delta = 0$ was assumed, the values of the combined moment derivatives were reasonable. The discrepancy in the results can be explained by the fact that $d\delta$ is such a small number that a very slight error in magnitude may be large percentage wise. In

the solution of the simultaneous equations the $d\delta$ term must be eliminated in order to obtain the combined derivatives. As this process is dependent upon either a division or a multiplication, it is the percentage error that carries through. Also, as the entire solution depends upon the subtraction of relatively large numbers to obtain a small number, a large percentage error in $d\delta$ causes completely erroneous results. Therefore the assumption of $d\delta = 0$ was used to calculate the tabulated results.

The individual stability derivatives are in close agreement with those previously obtained although both C_{m_δ} and $C_{m_{d\theta}}$ are somewhat high. In the calculation of the moment derivatives, with the exception of C_{m_δ}, the value of $C_{m_{d\theta_1}}$ was found to be important. (This applies especially to the impulse data and is brought out later in this paper.) The previous results obtained from oscillation data indicate that $C_{m_{d\theta_1}}$ is very close to zero. In order to determine the effect the magnitude of $C_{m_{d\theta_1}}$ has on the values of the moment derivatives, $C_{m_{d\theta_1}}$ was assumed to equal zero. The table below indicates these results.

	$C_{m_{d\theta_1}} = -0.0102$	$C_{m_{d\theta_1}} = 0$
C_{m_δ}, per rad.	−0.962	−0.963
C_{m_α}	−0.324	−0.339
$C_{m_{d\alpha}}$	−0.023	−0.028
$C_{m_{d\theta}}$	−0.044	−0.039
$C_{m_{i_t}}$	−2.44	−2.82
$d\varepsilon/d\alpha$	0.683	0.719

In general the results obtained from the step function data are considered to be good representative values of the stability derivatives of the F-80A aircraft.

Response to the Impulse Deflection of the Elevator

The values of $C_{L_\alpha} = 5.91$ and $C_{L_{\alpha_1}} = 6.37$ do not agree with either previous results or theory. These results were obtained from an analysis of Points 2 through 20, excepting Point 10. As the difference between C_{L_α} and $C_{L_{\alpha_1}}$ is due to the tail lift, it would be expected that C_{L_α} would be greater than $C_{L_{\alpha_1}}$. A possible explanation might be that the poor pitching velocity data caused the discrepancy. As the difference between C_{L_α} and $C_{L_{\alpha_1}}$ should be very small, an average value of $C_{L_\alpha} = C_{L_{\alpha_1}} = 6.13$ was assumed for use in subsequent calculations.

The values of h and the "tail-off" derivatives were found to vary somewhat from the results obtained by Cornell Aeronautical Laboratory, Inc. In particular $h = 0.010$ was obtained as compared to $h = 0.0133$ obtained previously, and $h = 0.015$ as determined from analysis of the step data. $C_{m_{d\theta_1}}$, in this case, changed sign to $C_{m_{d\theta_1}} = 0.0176$. This value, although well within the range of the scatter obtained by frequency response analyses, causes some trouble in the determination of the actual derivatives. This point is discussed later in this section at somewhat greater length.

In the case of the impulse, as well as with the step, small

terms were troublesome. A first attempt to analyze the data at points evenly divided throughout the range (even numbered points 2 through 20) ended in absurd answers for the combined constants. However, by making the assumption that the lift due to the deflection of the elevator as negligible, the tabulated results were obtained. The above assumption is reasonable because the elevator is deflected for a very short time only. The use of the assumption in effect causes $d\delta = \dot{\delta} = 0$ for all points, and thus eliminates the difficulties of small numbers, as previously discussed.

The effect of $C_{m_{d\theta_1}}$ on the calculations of the stability parameters is quite pronounced. If, as was done in the consideration of the step function, $C_{m_{d\theta_1}}$ is assumed to be $C_{m_{d\theta_1}} = 0$, the derivatives change in the manner indicated below:

	Previous Results	$C_{m_{d\theta_1}} = 0$	$C_{m_{d\theta_1}} = 0.0176$
C_{m_δ}, per rad.	-0.86	-0.86	-0.86
C_{m_α}, per rad.	-0.35	-0.331	-0.357
$C_{m_{d\alpha}}$, per rad.	-0.022	-0.025	-0.0335
$C_{m_{d\theta}}$, per rad.	-0.032	-0.0354	-0.0268
$C_{m_{i_t}}$, per rad.	-2.44	-2.46	-3.08
$d\varepsilon/d\alpha$	0.68	0.704	0.756

The above table indicates that possibly the value of $C_{m_{d\theta_1}}$ should be closer to zero. At any rate it is evident that the success in the method of analysis used is dependent upon the accuracy of $C_{m_{d\theta_1}}$.

A comparison of the results obtained from the transient data with those previously obtained from frequency response data indicates that possibly the step function data is best suited for this type of analysis. This result is as expected, as the equations used depended to a great extent upon the values of integrals. The step function data yielded larger integrals, and the errors in measurement are smaller percentage wise. However, if the equations of Appendix D are utilized, the impulse may be as good as the step function.

Another consideration might be that the frequency content of a step function is quite low as compared to that of an impulse in the high frequency range. This has the advantage that any higher order effects (important at high frequencies) that were neglected from the equations of motion will be unimportant. However, the frequency content of a step function is very high in the low frequency range. As phugoid frequencies are also low, precautions must be observed to see that the assumption of constant airspeed is valid.

CONCLUSIONS AND RECOMMENDATIONS

The results of the present investigation lead to the following conclusions:

1. It is feasible to obtain the longitudinal stability derivatives of an airplane by the "Equations of Motion Method of Analysis of Transient Response Data."

2. The step function data yields better results than impulse data if the equations used depend upon integrals, as they did in this study. This is not necessarily true if variations of the equations of motion are used in which integrals are not involved.

3. With good instrumentation it should be possible to obtain the inertia parameter h, and thus the moment of inertia of the airplane by transient response analysis.

4. Reasonable values of the longitudinal derivatives of the F-80A for a Mach Number of M = 0.707 and a cg position of 27% MAC are:

C_{L_α}	6.22 / radian
C_{m_δ}	-0.96 / radian
C_{m_α}	-0.324 / radian
C_{md_α}	-0.023 / radian
C_{md_θ}	-0.044 / radian

5. The use of the "Equations of Motion Method" is

dependent upon small differences of large numbers so that variables of very small magnitude must be avoided where possible.

6. The accurate evaluation of $C_{m_{d\theta_1}}$ is very important in the subsequent calculation of the moment derivatives in the method used in this investigation.

Recommendations:

1. Additional flight data should be analyzed by the method outlined in this report in order to develop the method further. This additional data should include information at various Mach Numbers and center of gravity positions.

2. The equations outlined in Appendix D should be utilized, provided the instrumentation gives sufficiently accurate pitching velocity response.

REFERENCES

1. H. Greenberg. A Survey of Methods for determining Stability Parameters of an Airplane from Dynamic Flight Measurements. National Advisory Committee for Aeronautics Technical Note No. 2340 dated April, 1951.

2. E. Seckel. Discussion of Dynamic Stability Flight Data Analysis Procedures Suggested by R. S. Swanson and NACA Ames Laboratory Personnel. Cornell Aeronautical Laboratory Flight Research Memorandum No. 97 dated 10 February, 1950.

3. R. C. Kidder. Dynamic Longitudinal Stability Flight Tests of an F-80A Airplane by the Forced Oscillation and Step Function Response Methods Including Measured Horizontal Tail Loads. Cornell Aeronautical Laboratory Report No. TB 495-F-11 dated 10 February, 1950.

4. E. Seckel. Suggested Methods of Analyzing Airplane Longitudinal Frequency Response Data, Including a Method (Utilizing Observed Tail Loads) for Resolving the Lumped Transfer Constants b_o and k_o, into their Aerodynamic Components. Cornell Flight Research Memorandum No. 75 dated 12 August, 1949, revised 17 November, 1949.

TABLE I

DIMENSIONS AND LEADING PARTICULARS

F-80A Serial No. 485169

I. Principal Dimensions	Dimensions
A. Airplane - General	
Span	38' $10\frac{1}{2}$ in.
Length (overall)	34' 6 in.
Height	11' 4 in.
B. Wing	
Aspect Ratio	6.38
Mean Aerodynamic Chord (MAC)	80.6 in.
C. Empennage	
Tail length, l_t,	14.87 ft.
Tail length, l,	15.338 ft.
Aspect ratio	5.56
Root chord	4.33 ft.
Tip chord	1.583 ft.
Elevator hinge center line	75% chord
II. Areas	
A. Wing	
Total wing area	237.6 sq. ft.
B. Empennage	
Total horizontal tail area	43.5 sq. ft.

TABLE I
(continued)

	Dimensions
III. Mass Characteristics	
A. Weight of aircraft	
Run 5301	10,315 lb.
Run 5068	10,263 lb.
B. Moment of inertia about Y axis	
Run 5301	15,130 slug ft.^{2}
Run 5068	15,130 slug ft.^{2}
C. C.G. Position, % MAC	
Run 5301	27%
Run 5068	27%
D. Weight of tail outboard of strain gage installation, w_t	100 lb.
IV. Oscillograph Trace Sensitivities	
(Same for both 5068 and 5301)	
δ	0.425 deg. / in.
q	4.39 deg. / sec. / in.
n_z	0.354 g / in.
M_t	151 lb. / in.
V. Other Particulars	
A. Run 5301, Step function data	
V_o	726 ft. / sec.
ρ	0.001271 slugs / ft.^{3}
Mach Number	0.704
Altitude	20,160 ft.

TABLE I
(continued)

	Dimensions
τ	1.49 sec.
μ	160.8
C_L	0.132

B. Run 5068, Impulse function data

V_o	733.5 ft. / sec.
ρ	0.001258 slugs / ft.^{3}
Mach Number	0.708
Altitude	20,230 ft.
τ	1.452 sec.
μ	158.6
C_L	0.1278

INDEX OF FIGURES

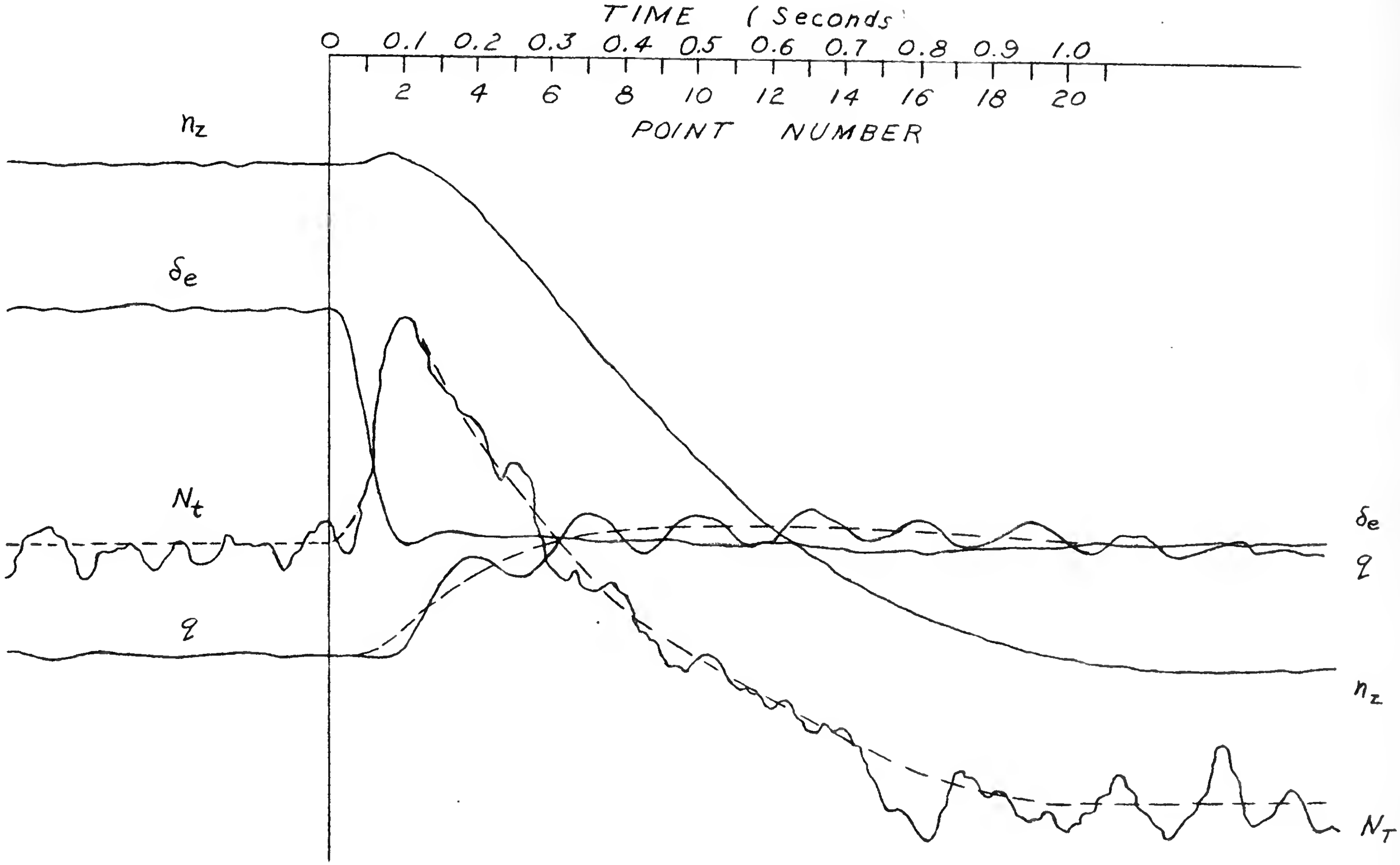

FIG. 1. STEP FUNCTION RESPONSE
DATA ---- RUN 5301
F-80A

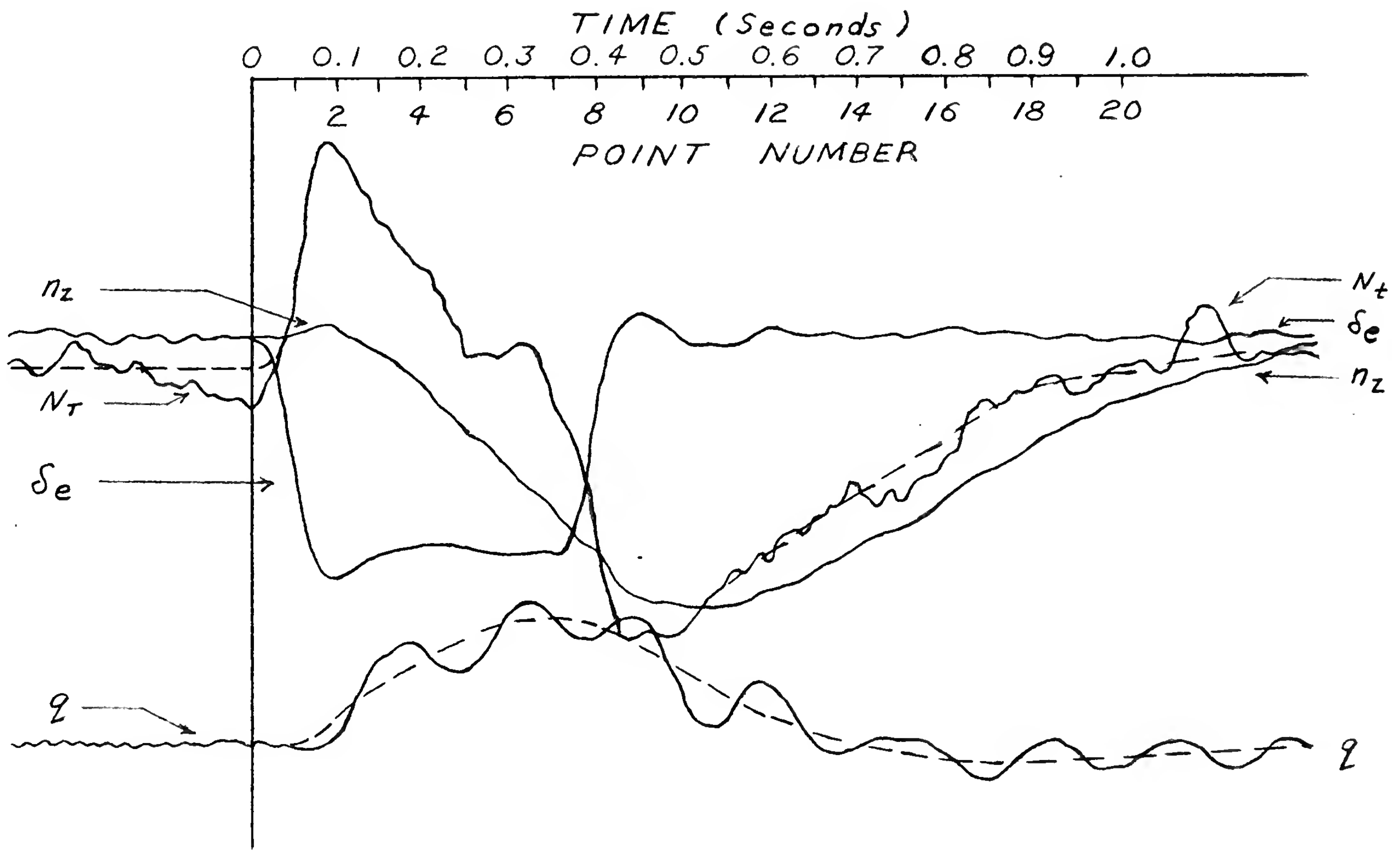

FIG. 2. IMPULSE FUNCTION RESPONSE
DATA - - - - - RUN 5068
F-80A

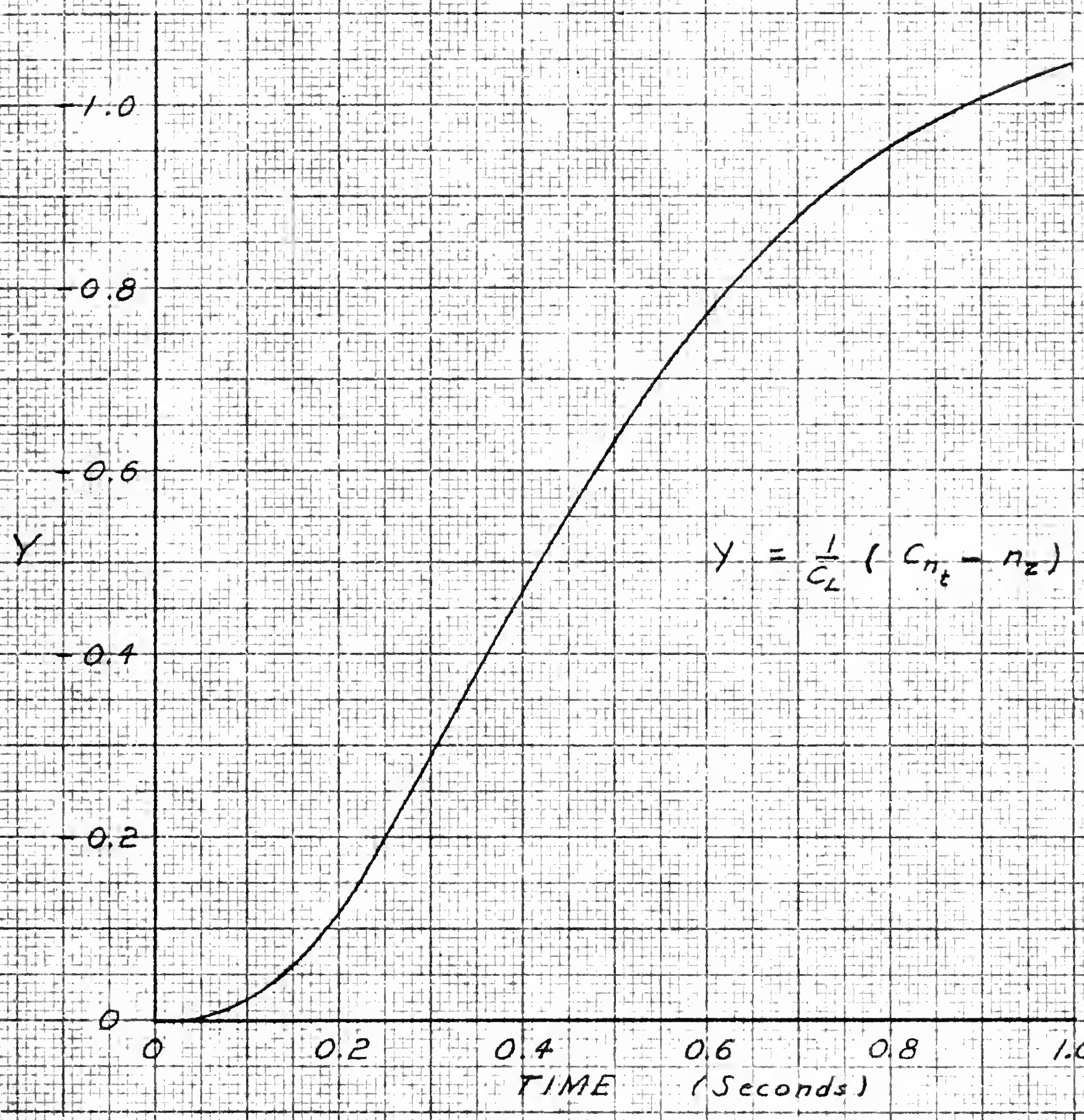

FIG. 3 VARIATION OF "Y" WITH TIME RUN 5301

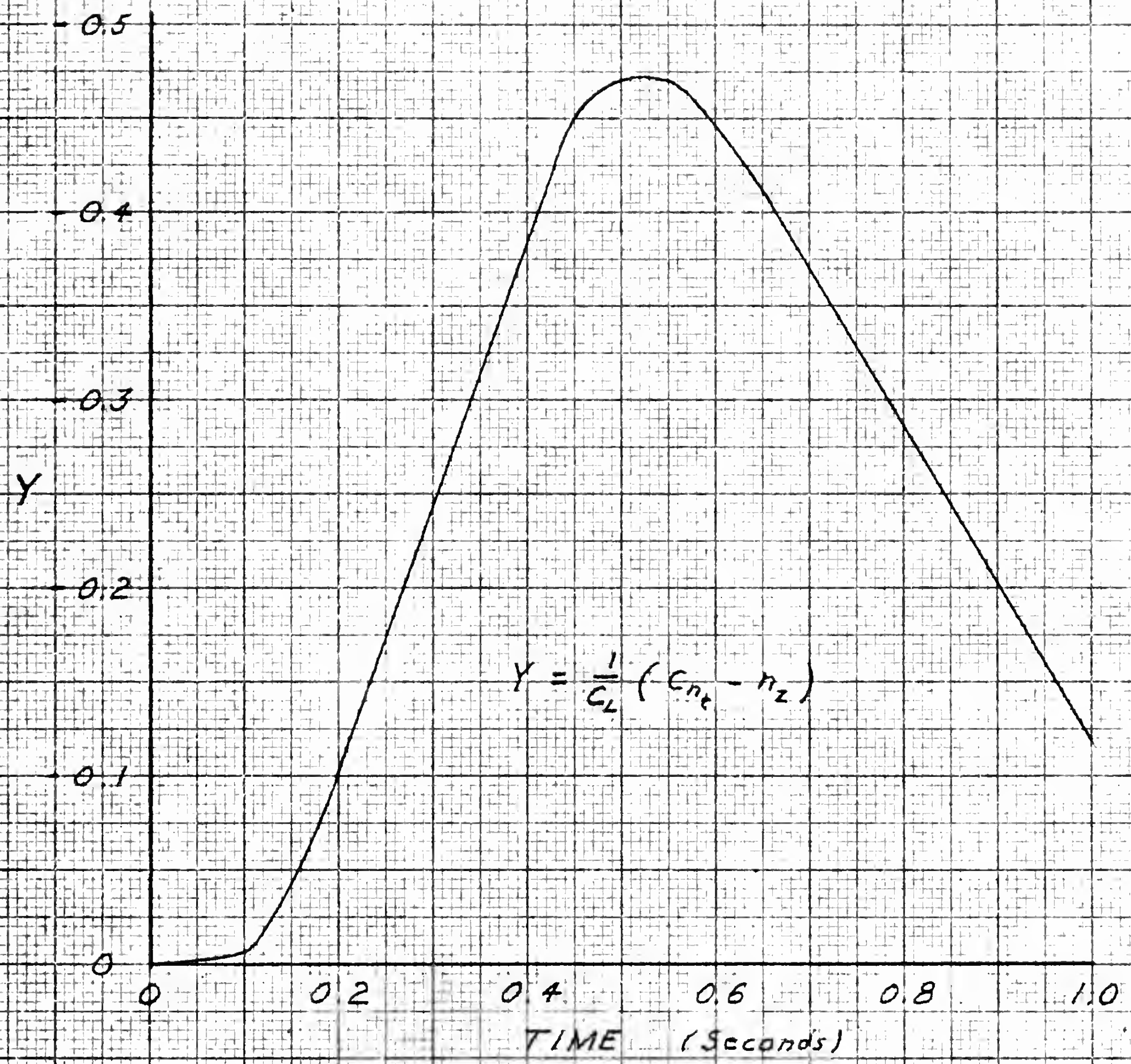

FIG. 4. VARIATION OF "Y"
WITH TIME
RUN 5068

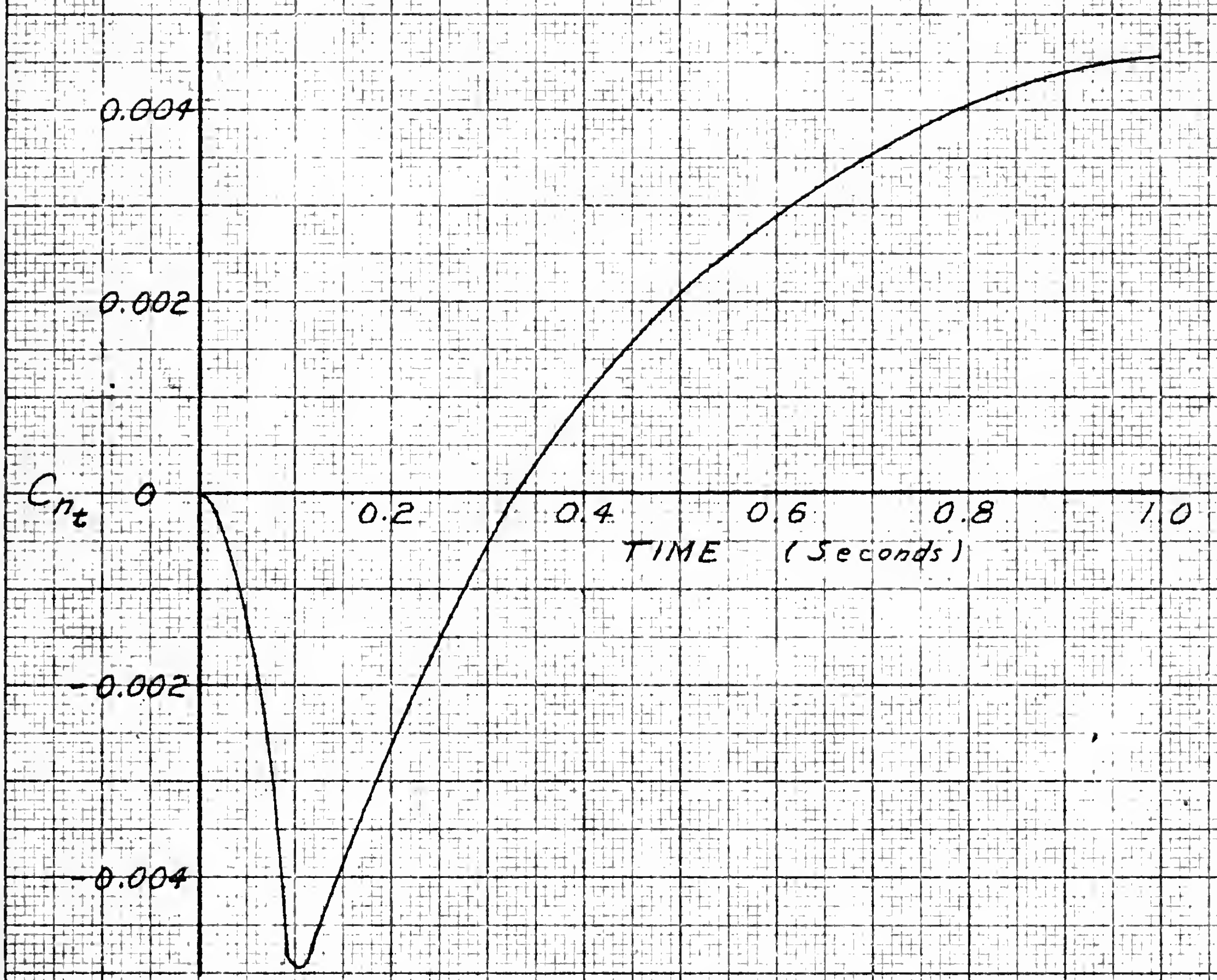

FIG. 5. VARIATION OF TAIL LOAD COEFFICIENT WITH TIME
RUN 5301

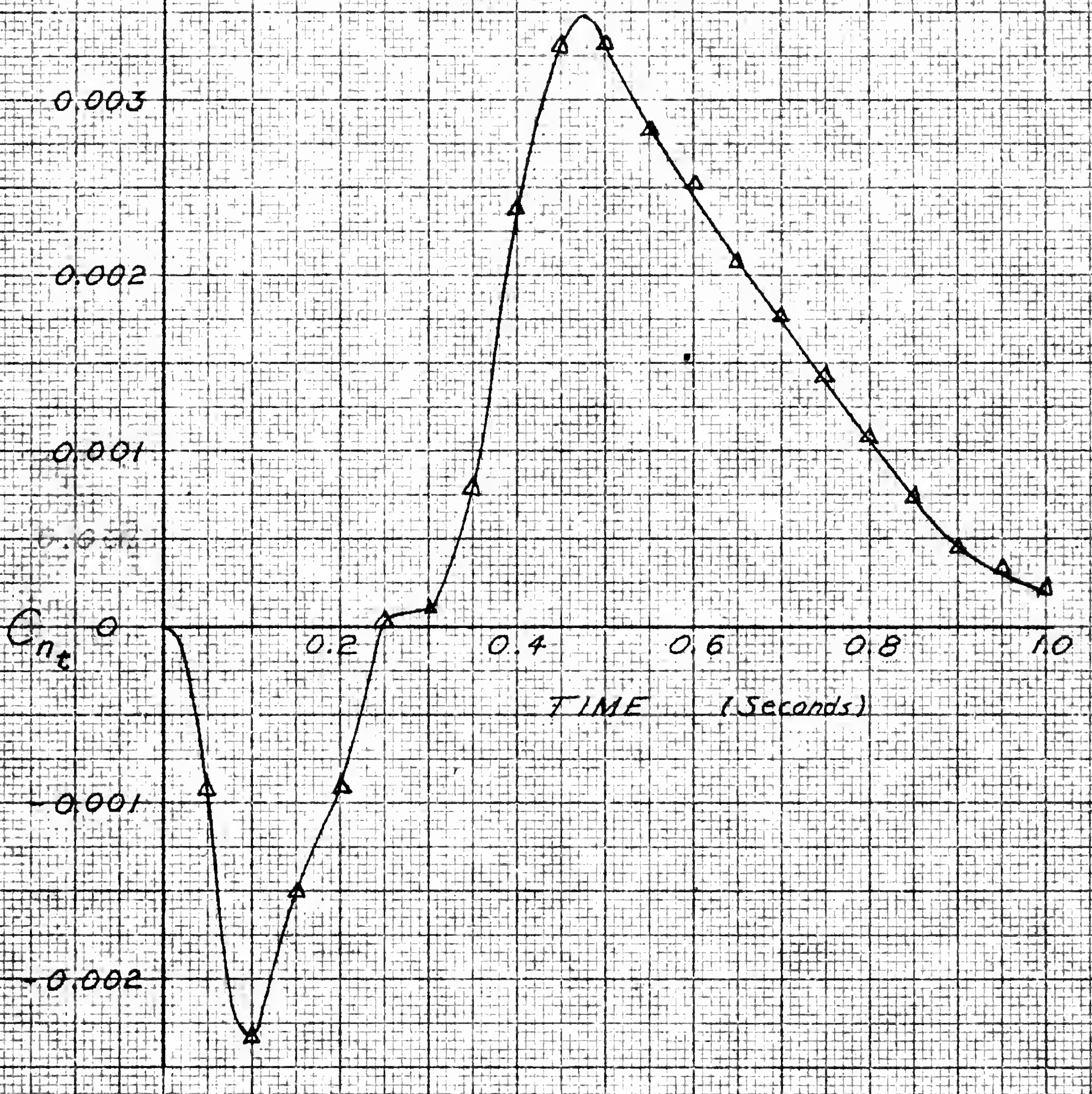

FIG. 6. VARIATION OF TAIL LOAD COEFFICIENT WITH TIME RUN 5068

APPENDIX A

A METHOD FOR CONVERTING RATE GYRO FREQUENCY RESPONSE CALIBRATION DATA TO A FORM APPLICABLE TO TRANSIENT INPUTS

Theory

Because of the fact that the rate gyro used to measure the pitching velocity was calibrated with a sine wave input, it was necessary to convert the calibration data in such a manner that it could be used for transient inputs. The frequency response calibration data are presented in Fig. 1A as amplitude rate and phase angle versus frequency curves.

It is assumed that the rate gyro is essentially a second order system and can be defined by the equation

$$(1) \qquad q = \frac{1}{w_n^2}\ddot{q}_o \quad \frac{2\zeta}{w_n}\dot{q}_o + q_o$$

or

$$(2) \qquad q = C\left(X + \frac{2\zeta}{w_n}\dot{X} \quad \frac{1}{w_n^2}\ddot{X}\right)$$

where

w_n = natural frequency of the system

ζ = the damping ratio

q_o = pitching velocity indicated by the rate gyro, radians / sec.

q = pitching velocity of airplane, radians / sec.

C = sensitivity of the rate gyro, radians / sec. / in.

X = displacement of oscillograph record, in.

Therefore if ζ and w_n for the system are known, it is possible to determine the pitching velocity of the airplane. In the present

analysis Equation (2) was integrated and the integral of q was obtained. This was done in order to avoid taking second slopes of the recorded data.

$$(3) \quad \int q\,dt = \frac{1}{w_n^2}\dot{q}_o + \frac{2\xi}{w_n} q_o + \int q_o\,dt$$

If Equation (1) is transposed to operator form and solved for the amplitude ratio of q_o / q the following equation is obtained:

$$(4) \quad AR\ (s) = \frac{q_o\ (s)}{q\ (s)} = \frac{1}{\frac{s^2}{w^2} + \frac{2\xi s}{w_n} + 1}$$

Then by applying the well known "jw" substitution for the operator "s" the equation becomes:

$$(5) \quad AR(jw) = \frac{1}{(1 - \frac{w^2}{w_n^2}) + (2\xi\frac{w}{w_n})\ j}$$

and

$$(6) \quad \left| AR \right| = \frac{1}{\sqrt{(1 - \frac{w^2}{w_n^2})^2 + (2\xi\frac{w}{w_n})^2}}$$

The phase angle, ϕ, is determined as:

$$(7) \quad \phi = - \text{arc ton}\ \frac{2\xi\frac{w}{w_n}}{1 - (\frac{w}{w_n})^2}$$

The damping ratio, ξ, for the system can be obtained by comparing the plot of amplitude ratio, AR, versus phase angle, ϕ,

to constant damping ratio curves of a standard second order system (Fig. 2A). If constant damping ratio curves are not readily available, they can easily be plotted from Equations 6 and 7 by assuming values of w/w_n and solving for the corresponding values of AR and ϕ at the desired damping ratio.

Fig. 2A indicates that the damping ratio for the present system is approximately $\zeta = 0.15$. Because of the wide scatter of points it is difficult to fair the data. However, as the pitching velocity correction is small, a small error in ζ does not materially affect the resulting pitching velocity of the airplane.

As ζ is now known, it is possible to assume a value of w_n and to determine both AR and ϕ for each of several frequencies (Equation 6).

A comparison of the values of AR and ϕ to points on the curve in Fig. 1A indicates whether the assumed value of w_n is the correct one. In the present case $w_n = 21.2$ fits the curve quite well (Fig. 2A).

If the values of $\zeta = 0.15$, $w_n = 21.2$ radians / sec., and $C = 0.0767$ radians / sec. / in. are substituted into Equation (2) and the equation integrated, the following expression for the correction of the rate gyro reading is obtained:

$$(8) \quad \int q\,dt = 0.0767 \int x\,dt + 0.001085X + 0.0001702\dot{X}$$

A plot of $\int q\,dt$ versus time is shown in Fig. 3A. The final values of q were found by reading the slopes of this curve at the points desired in the calculations.

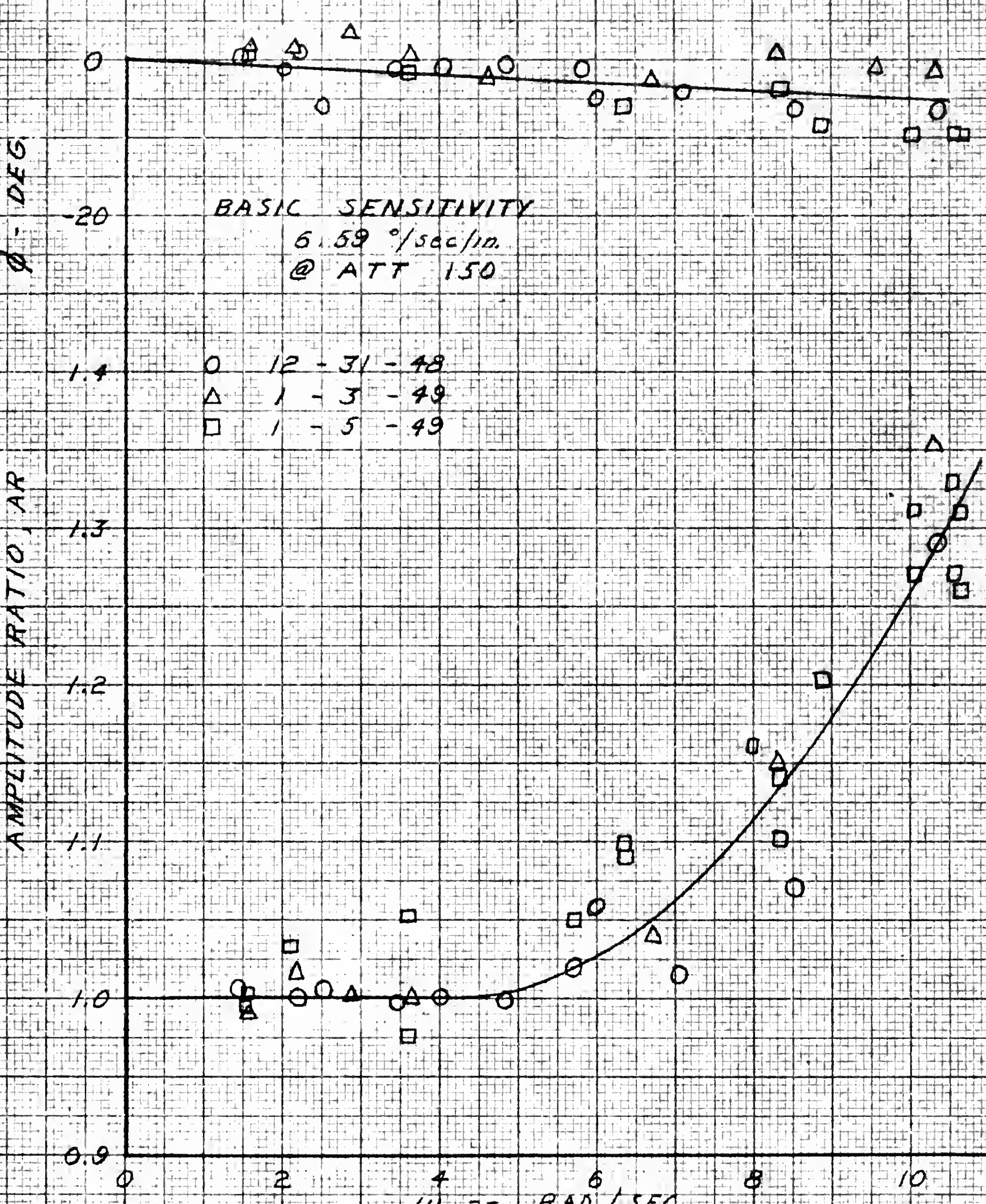

FIG. 1A. UNDAMPED RATE GYRO FREQUENCY RESPONSE PENDULUM TESTS

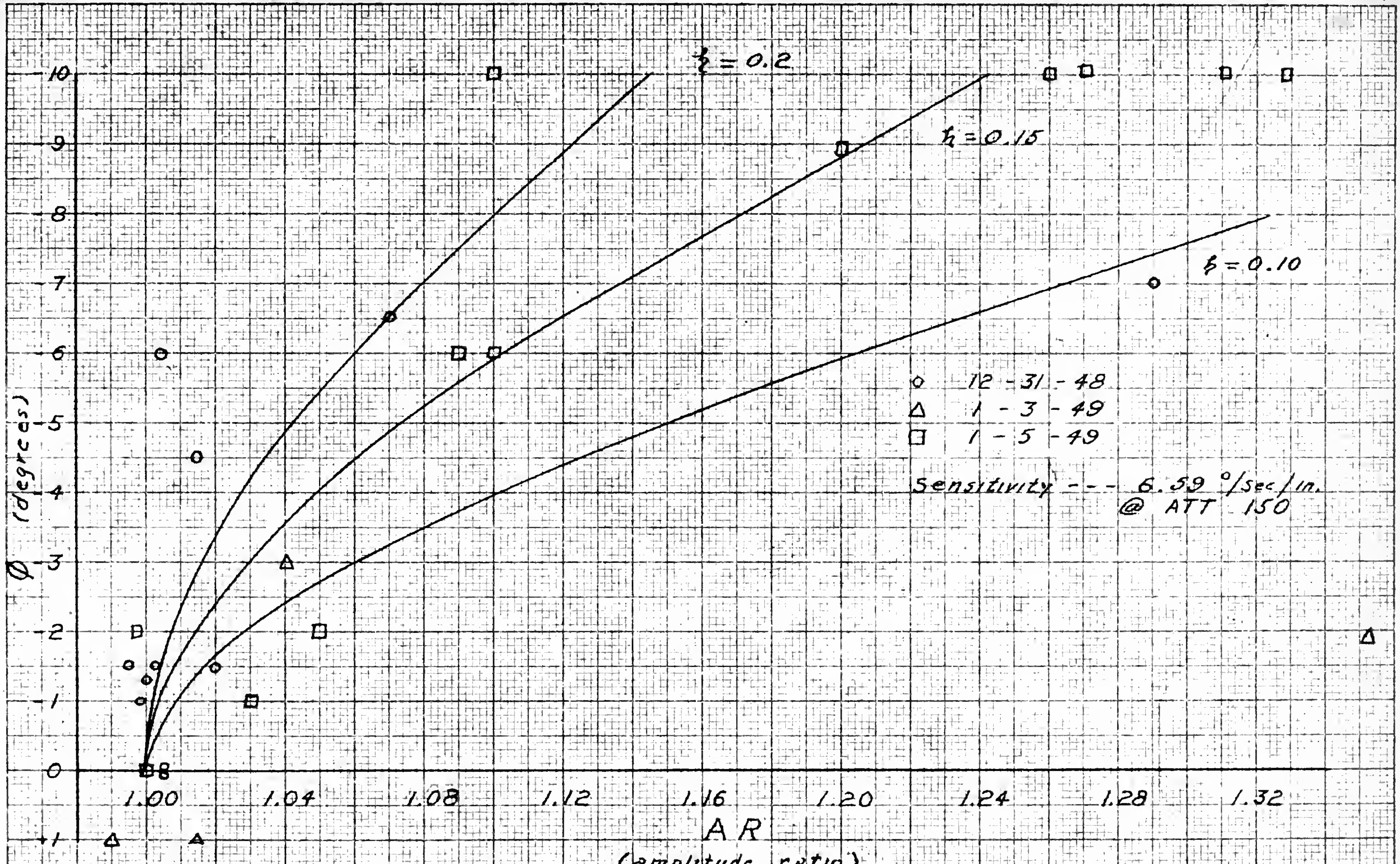

FIG. 2A. · GRAPHICAL DETERMINATION OF RATE GYRO DAMPING CONSTANT

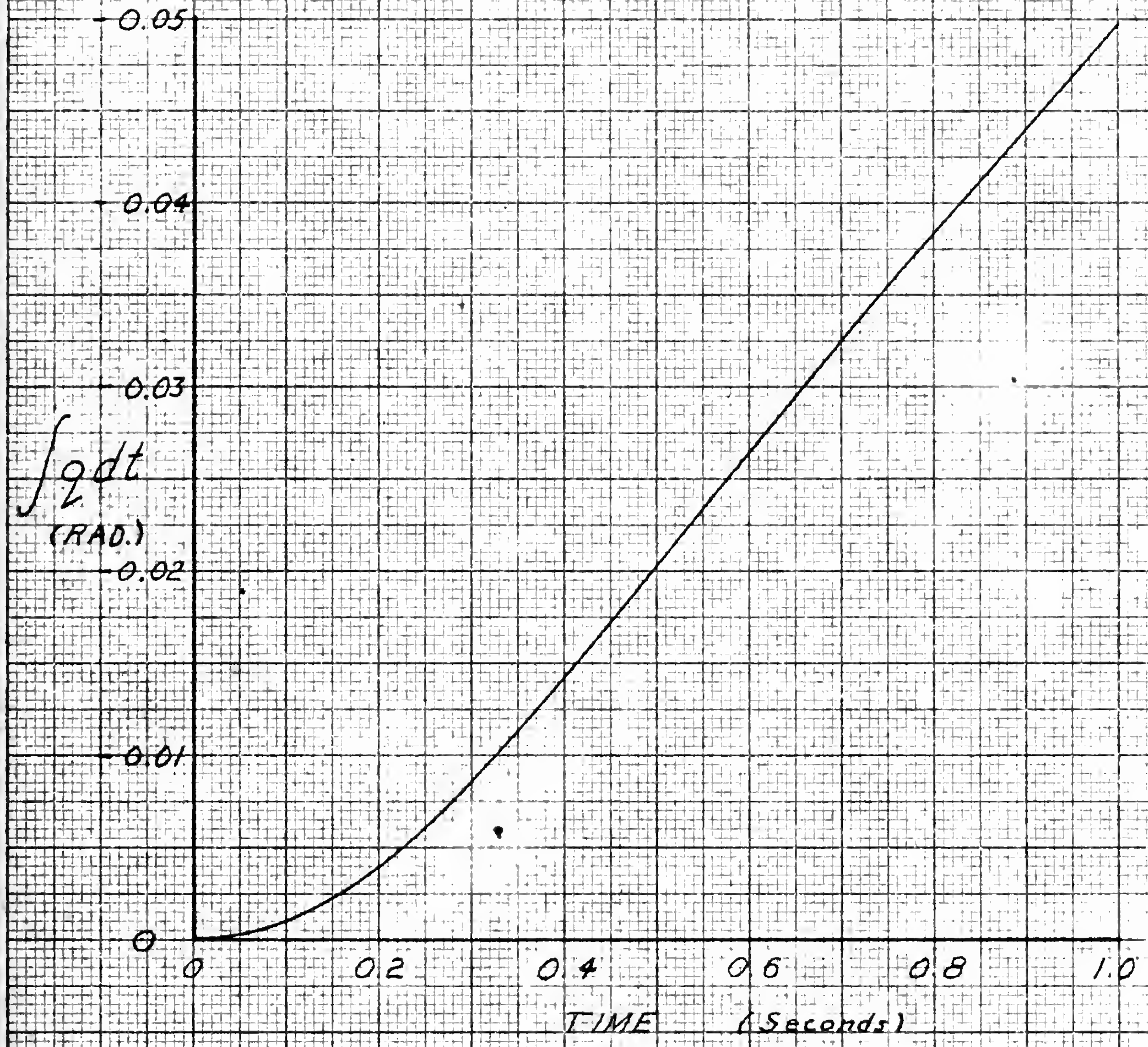

FIG. 3A. VARIATION OF $\int q\,dt$ WITH TIME
RUN 5301

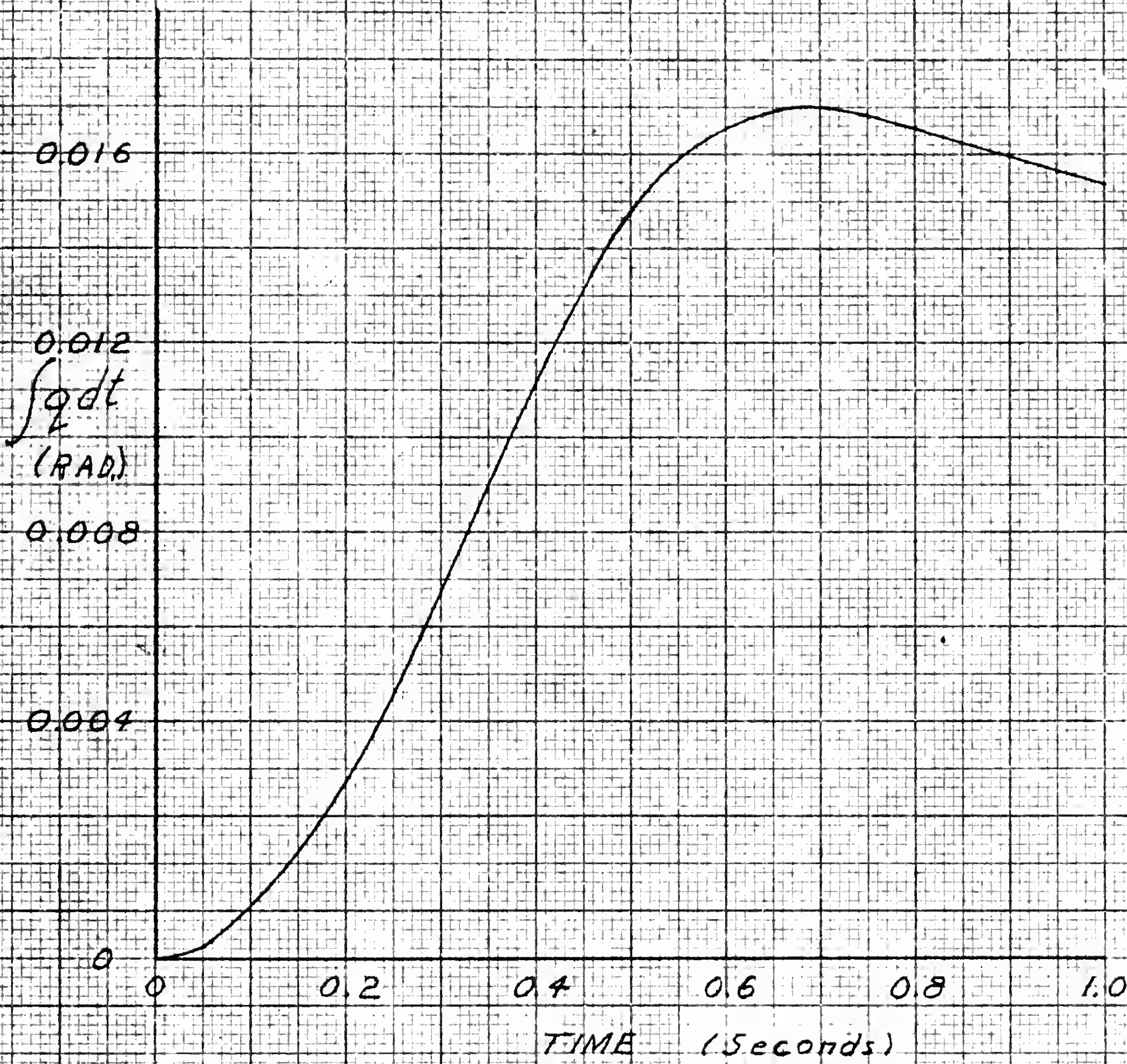

FIG. 4 A. VARIATION OF $\int q\,dt$
WITH TIME
RUN 5068

APPENDIX B

HORIZONTAL TAIL LOAD CORRECTION FOR INERTIA LOADS

The tail load response data shown in Fig. 1 and 2 were measured by a strain gage. The response shown is composed of both aerodynamic loads and the inertia loads of the stabilizer outboard of the strain gage installation. The measured tail load, $(N_t)_m$, must be corrected as follows:

$$(1) \quad N_t = (N_t)_m - W_t \left[- n_z - \frac{l\dot{q}}{g} \right]$$

where W_t = weight of tail outboard of strain gage fittings.

For the F-80A strain gage installation, W_t = 100 lbs. and l = 15.338 ft. Hence Equation (1) becomes

$$N_t = (N_t)_m + 100 n_z + 47.6\, \dot{q}$$

APPENDIX C

SAMPLE CALCULATIONS

A complete outline of the calculations for the stability derivatives for the case of the elevator step-function input (Run 5301) is presented below.

The values of the basic variables, for any one instant, were taken from Fig. 1 and are listed in Tables I E - IV E. All measurements of the basic variables were determined within a period of one second after the elevator deflection was applied. The variable q was corrected for instrument calibration error as outlined in Appendix A. The variable H_t was corrected for the presence of horizontal tail inertia loads as outlined in Appendix B.

The slope of the lift curve, C_{L_α}, was determined as follows:

$$\frac{C_{L_\alpha}}{C_L} v + dn_z - \frac{C_{m\,\delta}}{l_t/c} \frac{1}{C_L} d\delta = 0$$

where

$$v = \frac{C_L}{2} n_z + \tau q$$

Applying the method of least squares and assuming that the greatest error lies in v, the normal equations are

$$\frac{C_L}{C_{L_\alpha}} \sum (dn_z)^2 - \frac{C_{m\,\delta}}{l_t/c\; C_{L_\alpha}} \sum d\delta \cdot d_{n_z} + \sum v \cdot d_{n_z} = 0$$

$$\frac{C_L}{C_{L_\alpha}} \sum d\delta \cdot d_{n_z} - \frac{C_{n\,\delta}}{l_t / c \; C_{L_\alpha}} \sum (d\delta)^2 + \sum v \cdot d\delta = 0$$

The numerical values for the summations indicated from Table V E can now be substituted and the equations solved for C_L / C_{L_α}.

$$C_L / C_{L_\alpha} = 0.021228$$

since $\quad C_L = 0.132$

then $\quad C_{L_\alpha} = 6.22$ per radian

The slope of the lift curve for the "tail-off" condition, $C_{L_{\alpha_1}}$, was determined as follows:

$$\frac{C_L}{C_{L_{\alpha_1}}} dy = \tau q + \tfrac{1}{2} C_{n_t} - \frac{C_L}{2} y$$

where $\quad y = \left(\frac{1}{C_L} C_{n_t} - n_z \right)$

A graph of the y variable versus time is shown in Fig. 3. This graph was used for determining the values of dy for any one instant (or point).

Applying the method of least squares and assuming that the greatest error lies in τq, the normal equation becomes

$$\sum dy - \tau q - \frac{C_L}{C_{L_{\alpha_1}}} \sum (dy)^2 + \sum dy \cdot (\tfrac{1}{2} C_{n_t}) - \sum dy \cdot (\frac{C_L}{2} y) = 0$$

The numerical values for the summations indicated from Table VI E were substituted and the equation solved for $C_L / C_{L_{\alpha_1}}$.

$$C_L / C_{L_{\alpha_1}} = 0.022261$$

$$C_{L_{\alpha_1}} = 5.93 \text{ per radian}$$

The values of h, $C_{m_{\alpha_1}}$, and $C_{m_{d\theta_1}}$ were determined as follows:

$$\left(C_{m_{\alpha_1}} + \frac{C_{L_{\alpha_1}}}{2} C_{m_{d\theta_1}}\right) \cdot \int yd\ t/\tau + \left(C_{m_{d\theta_1}} - h \frac{C_{L_{\alpha_1}}}{2}\right) y$$

$$- hw = \frac{-C_{L_{\alpha_1}}}{C_L} \frac{1}{c} \int C_{n_t}\ d\ t/\tau$$

where $\quad y = \dfrac{1}{C_L} C_{n_t} - n_z$

$$w = dy - \frac{C_{L_{\alpha_1}}}{2\ C_L} C_{n_t}$$

Graphs of the variables y and C_{n_t} are shown in Fig. 3 and 5, respectively. These graphs were used in determining the integrals of the variables.

Putting the above equation in the form

$$A \int yd\ t/\tau + By - hw = - C \int C_{n_t}\ d\ t/\tau$$

and applying the method of least squares, assuming that the greatest error lies in w, the normal equations become:

$$\sum w \cdot \int C_{n_t}\ d\ t/\tau + \frac{C}{h} \sum (C_{n_t}\ d\ t/\tau)^2 - \frac{A}{h} \sum \int yd\ t/\tau \cdot$$

$$\int C_{n_t}\ d\ t/\tau - \frac{B}{h} \sum y \cdot \int C_{n_t}\ d\ t/\tau = 0$$

$$\sum w \cdot \int yd\ t/\tau + \frac{C}{h}\sum \int yd\ t/\tau \cdot \int C_{n_t}\ d\ t/\tau - \frac{A}{h}\sum\left(\int yd\ t/\tau\right)^2$$

$$- \frac{B}{h}\sum y \cdot \int yd\ t/\tau = 0$$

$$\sum w \cdot y + \frac{C}{h}\sum y \cdot \int C_{n_t}\ d\ t/\tau - \frac{A}{h}\sum y \cdot \int yd\ t/\tau$$

$$- \frac{B}{h}\sum y^2 = 0$$

The numerical values for the summations indicated from Tables VII E - IX E were substituted in the equations and the equations solved for

$$C/h = -\ 6{,}644.7$$

$$A/h = 30.4466$$

$$B/h = -\ 3.72994$$

where

$$C = -\ \frac{C_{L_{\alpha_1}}}{C_L}\ \frac{1}{c} = -\ \frac{5.93}{0.132}\ (2.22) = -\ 99.7224$$

therefore, $h = 0.01501$.

The value of h determined from ground test of the inertia of the airplane was found to be $h = 0.0133$ and is used for all further calculations.

$C_{m_{d\theta_1}}$ and $C_{m_{\alpha_1}}$ are solved for as follows:

$$B = C_{m_{d\theta_1}} - h\ \frac{C_{L_{\alpha_1}}}{2} = -\ 3.72994\ h$$

$$C_{m_{d\theta_1}} - 0.0133 \left(\frac{5.93}{2}\right) = -3.72994 \, (0.0133)$$

$$C_{m_{d\theta_1}} = -0.0102 \text{ per radian}$$

$$A = C_{m\alpha_1} + \frac{C_{L\alpha_1}}{2} C_{m_{d\theta_1}} = 30.4466 \, h$$

$$C_{m\alpha_1} + \frac{5.93}{2} (-0.0102) = 30.4466 \, (0.0133)$$

$$C_{m\alpha_1} = 0.43518 \text{ per radian}$$

The values of the combined moment derivative coefficients were determined as follows:

$$\left(C_{m_\alpha} + \frac{C_L}{2} C_{m_{d\theta}}\right) \int n_z \, d\, t/\tau \quad \left(C_{m_{d\theta}} - h \frac{C_{L_\alpha}}{2}\right) n_z$$

$$- \frac{C_{m_\delta}}{l_t/c} \frac{1}{C_L} \left(C_{m_\alpha} + C_{L_\alpha} \frac{l_t}{c}\right) \int \delta \, d\, t/\tau$$

$$- \frac{C_{m_\delta}}{l_t/c} \cdot \frac{1}{C_L} \left(C_{m_{d\alpha}} + C_{m_{d\theta}}\right) \delta + \frac{C_m}{l_t/c} \frac{h}{C_L} d\delta$$

$$= h d n_z$$

For the step function elevator deflection the derivative $d\delta$ was taken as $d\delta = 0$ at the points in question.

Putting the above equation in the form

$$D \int n_z \, d\, t/\tau + E \, n_z - F \int \delta \, d\, t/\tau - G\delta = h d n_z$$

and applying the method of least squares, assuming that the greatest error lies in dn_z, the normal equations become

$$\sum hdn_z \cdot \int n_z d\, t/\tau - D \sum \left(\int n_z d\, t/\tau \right)^2 - E \sum n_z \cdot \int n_z\, d\, t/\tau$$

$$+ F \sum \int \delta\, d\, t/\tau \cdot \int n_z d\, t/\tau + G \sum \delta \cdot \int n_z\, d\, t/\tau = 0$$

$$\sum n_z \cdot hdn_z - D \sum n_z \cdot \int n_z\, d\, t/\tau - E \sum (n_z)^2 + F \sum n_z \cdot \int \delta\, d\, t/\tau$$

$$+ G \sum n_z \cdot d\delta = 0$$

$$\sum hdn_z \cdot \int \delta\, d\, t/\tau - D \sum \int n_z\, d\, t/\tau \cdot \int \delta\, d\, t/\tau - E \sum n_z \cdot \int \delta\, d\, t/\tau$$

$$+ F \sum \left(\int \delta\, d\, t/\quad \right)^2 + G \sum \delta \cdot \int \delta\, d\, t/\tau = 0$$

$$\sum \delta \cdot hdn_z - D \sum \delta \cdot \int n_z\, d\, t/\tau - E \sum \delta \cdot n_z + F \sum \delta \cdot \int \delta\, d\, t/\tau$$

$$+ G \sum (\delta)^2 = 0$$

After substituting the numerical value for the summations as indicated from Tables X E - XIV E, the following constants were determined:

$$D = -0.461 = C_{n_\alpha} + \frac{C_L}{2} C_{n_{d\theta}}$$

$$E = -0.108 = C_{n_{d\alpha}} + C_{n_{d\theta}} - h \frac{C_{L_\alpha}}{2}$$

$$F = -44.25 \frac{C_{m_\delta}}{l_t/c} \frac{1}{C_L} \left(C_{m_\alpha} + C_{L_\alpha} \frac{l_t}{c} \right)$$

Five of the dynamic stability derivatives were solved from the simultaneous equations listed below.

$$C_{m_\alpha} = C_{m_{\alpha_1}} \; C_{m_{i_t}} \left(1 - \frac{d\epsilon}{d\alpha}\right)$$

$$C_{m_{d\theta}} = C_{m_{d\theta_1}} + \frac{1}{\mu} \frac{l_t}{c} C_{m_{i_t}}$$

$$C_{m_{d\alpha}} = \frac{1}{\mu} \frac{l_t}{c} C_{m_{i_t}} \frac{d\epsilon}{d\alpha}$$

$$C_{m_{d\alpha}} + C_{m_{d\theta}} - h \frac{C_{L_\alpha}}{2} = -0.108$$

$$C_{m_\alpha} + \frac{C_{L_\alpha}}{2} C_{m_{d\theta}} = -0.461$$

Substituting the known values in the above equations, they become

$$C_{m_\alpha} = 0.4352 + C_{m_{i_t}} \left(1 - \frac{d\epsilon}{d\alpha}\right)$$

$$C_{m_{d\theta}} = -0.0102 + 0.01381 \, C_{m_{i_t}}$$

$$C_{m_{d\alpha}} = 0.01381 \, C_{m_{i_t}} \frac{d\epsilon}{d\alpha}$$

$$C_{m_{d\alpha}} + C_{m_{d\theta}} = -0.0667$$

$$C_{m_\alpha} + 3.11 \, C_{m_{d\theta}} = -0.461$$

Solution of the simultaneous equations yields

$$C_{m_\alpha} = -\ 0.3242 \text{ per radian}$$

$$C_{m_{d\alpha}} = -\ 0.023 \text{ per radian}$$

$$C_{m_{d\theta}} = -\ 0.044 \text{ per radian}$$

$$C_{m_{i_t}} = -\ 2.44 \text{ per radian}$$

$$d\epsilon/d\alpha = 0.683$$

The value of C_{m_δ} was calculated from the solution of the combined moment derivative coefficient, F, determined previously.

$$\frac{C_{m_\delta}}{l_t/c} \quad \frac{1}{C_L} \quad \left(C_{m_\alpha} + C_{L_\alpha} \frac{l_t}{c}\right) = -\ 44.25$$

$$\frac{C_{m_\delta}}{2.22} \left(\frac{1}{0.132}\right) \left[0.324 + 6.22 \quad (2.22)\right] = -\ 44.25$$

$$C_{m_\delta} = -\ 0.962 \text{ per radian}$$

The solution of the stability derivatives in the case of the impulse-function elevator deflection was carried out in a similar manner.

APPENDIX D

VARIATION IN THE FORM OF THE EQUATIONS OF MOTION FOR DETERMINING DYNAMIC STABILITY DERIVATIVES

The use of the variable q was eliminated in this investigation wherever possible because of the poor rate gyro measurement of its transient response. However, by maximum use of q, the use of higher derivatives as variables in the moment equations can be avoided and the number of variables reduced.

Equation (8) in the THEORY AND ANALYSIS section of this report can be replaced by the equation

$$C_{m_{\alpha}} \frac{C_L}{C_{L_{\alpha_1}}} y + \tau C_{m_{d\theta_1}} q - \tau h dq - \frac{1}{c} C_{n_t} = 0$$

from which h, $C_{m_{\alpha_1}}$, and $C_{m_{d\theta_1}}$ can be readily calculated.

Equation (14) can also be replaced by the equation

$$\left(C_{m_{d\alpha}} \frac{C_L}{2} - C_{m_\alpha} \frac{C_L}{C_{L_\alpha}}\right) n_z + \left(\frac{C_{m_\delta}}{l_t/c} \frac{C_{m_\alpha}}{C_{L_\alpha}} + C_{m_\delta}\right) \delta$$

$$+ \left(C_{m_{d\alpha}} + C_{m_{d\theta}}\right) q - h \tau dq = 0$$

from which new combined moment derivative coefficients can be calculated.

If a good rate gyro is used to obtain the flight test response of q, these equations may give more accurate results than the equivalent equations that were utilized in this report.

APPENDIX E

TABLE IE

BASIC VARIABLES

Run 5301

Point Number	Time (sec.)	n_z(g)	dn_z	τq (rad.)	v^*
1	0.05	-0.01061	-0.739	-0.011921	-0.0126
2	0.10	-0.01417	+1.056	-0.0283	-0.0292
3	0.15	+0.03185	1.679	-0.04025	-0.0381
4	0.20	0.0956	2.376	-0.0589	-0.0526
5	0.25	0.1877	2.850	-0.0585	-0.04611
6	0.30	0.283	2.850	-0.0760	-0.0573
7	0.35	0.379	2.850	-0.0835	-0.0585
8	0.40	0.471	2.690	-0.0879	-0.0568
9	0.45	0.5595	2.585	-0.0909	-0.0540
10	0.50	0.641	2.265	-0.0954	-0.0531
11	0.55	0.720	2.110	-0.0954	-0.0479
12	0.60	0.791	1.951	-0.0954	-0.0432
13	0.65	0.850	1.635	-0.0954	-0.0393
14	0.70	0.904	1.424	-0.0954	-0.0358
15	0.75	0.946	1.187	-0.0909	-0.0284
16	0.80	0.985	1.003	-0.0849	-0.0198
17	0.85	1.012	0.792	-0.0849	-0.0181
18	0.90	1.038	0.739	-0.0849	-0.0173
19	0.95	1.060	0.528	-0.0835	-0.0135
20	1.0	1.080	0.369	-0.0820	-0.0107

* $v = \frac{C_L}{2} n_z + \tau q$

TABLE II E

BASIC VARIABLES

Run 5301

Point Number	Time (sec.)	$d\delta$	y^*	$(C_L/2)y$	$Cn_t/2$
1	0.05		-0.00163	-0.0001076	-0.000808
2	0.10		-0.02368	-0.001563	-0.002495
3	0.15		-0.06125	-0.00404	-0.00194
4	0.20	+0.00221	-0.11510	-0.00761	-0.001315
5	0.25	0.00221	-0.19980	-0.01320	-0.000798
6	0.30	0.00221	-0.28724	-0.01896	-0.000280
7	0.35	0.00221	-0.37597	-0.0248	+0.000200
8	0.40	0.00221	-0.46349	-0.0306	0.000495
9	0.45	0.00221	-0.54734	-0.0361	0.000802
10	0.50	0.00221	-0.62517	-0.0413	0.001045
11	0.55	0.00221	-0.70105	-0.0463	0.001250
12	0.60	0.00221	-0.76955	-0.0508	0.001415
13	0.65	0.00221	-0.82620	-0.0546	0.001570
14	0.70	0.00221	-0.87770	-0.0580	0.001733
15	0.75	0	-0.91715	-0.0606	0.001903
16	0.80	0	-0.95405	-0.0630	0.002045
17	0.85	-0.00221	-0.9796	-0.0647	0.002138
18	0.90	-0.00221	-1.0047	-0.0663	0.002200
19	0.95	-0.00221	-1.0258	-0.0675	0.002260
20	1.00	-0.00221	-1.0458	-0.0690	0.002261

$^*\ y = \frac{1}{C_L}\ C_{n_t} - n_z$

TABLE III E

BASIC VARIABLES

Run 5301

Point Number	Time (sec.)	dy	$\int y d\frac{t}{\tau}$	w^*	$\int Cn_t d\frac{t}{\tau}$	$\int q\, dt$
1	0.05	-0.3125	-0.00002015		-0.0000168	-0.0003150
2	0.10	-0.760	-0.0005062		-0.0001258	-0.0011215
3	0.15	-1.341	-0.0019162		-0.0001768	-0.002234
4	0.20	-1.878	-0.004866	-1.8189	-0.0002861	-0.003927
5	0.25	-2.655	-0.010066	-2.6192	-0.0003552	-0.005996
6	0.30	-2.655	-0.018126	-2.6416	-0.0003888	-0.008574
7	0.35	-2.655	-0.029206	-2.6640	-0.0003927	-0.011314
8	0.40	-2.655	-0.043296	-2.6773	-0.0003692	-0.014250
9	0.45	-2.460	-0.060386	-2.4960	-0.0003256	-0.017208
10	0.50	-2.232	-0.080146	-2.2790	-0.0002635	-0.020210
11	0.55	-2.085	-0.102366	-2.1412	-0.0001867	-0.023268
12	0.60	-1.848	-0.12697	-1.9116	-0.0000971	-0.023368
13	0.65	-1.640	-0.15372	-1.7106	0.0000051	-0.029418
14	0.70	-1.355	-0.18222	-1.4328	0.0001174	-0.032440
15	0.75	-1.191	-0.21237	-1.2766	0.0002403	-0.035432
16	0.80	-0.969	-0.24374	-1.0610	0.0003722	-0.038334
17	0.85	-0.819	-0.27624	-0.9150	0.0005115	-0.041266
18	0.90	-0.671	-0.30964	-0.7698	0.0006567	-0.044152
19	0.95	-0.581	-0.34384	-0.6825	0.0008059	-0.046941
20	1.0	-0.446	-0.37864	-0.5476	0.0009739	-0.049630

$$^* \quad w = dy - \frac{C_{L_{\alpha_1}}}{2C_L} C_{n_t}$$

TABLE IV E

BASIC VARIABLES

Run 5301

Point Number	Time (sec.)	δ (rad.)	$\int \delta \, d\, t/\tau$	hdn_z	$\int n_z d\, t/\tau$
2	0.10	0.01082	0.0003555	0.0141	-0.000712
4	0.20	0.01039	0.001057	0.0316	+0.001428
6	0.30	0.01039	0.001759	0.0379	0.01439
8	0.40	0.01061	0.002460	0.0358	0.03981
10	0.50	0.01068	0.003165	0.0301	0.07711
12	0.60	0.01068	0.003890	0.0260	0.12511
14	0.70	0.01103	0.004620	0.0189	0.18171
16	0.80	0.01111	0.005360	0.0133	0.24511
18	0.90	0.01090	0.006100	0.0098	0.31281
20	1.00	0.01090	0.006840	0.0049	0.38361

TABLE V E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR DETERMINATION OF C_{L_α}

Run 5301

Point Number	$(dn_z)(v)$	$vd\delta$	$(d\delta)(dn_z)$	$(d\delta)^2$	$(dn_z)^2$
4	-0.124954	-0.000116224	0.00525096	0.0000048841	5.64538
5	-0.131414	-0.000101903	0.00629850	0.0000048841	8.12250
6	-0.163305	-0.000126633	0.00629850	0.0000048841	8.12250
7	-0.166725	-0.000129285	0.00629850	0.0000048841	8.12250
8	-0.152792	-0.000125528	0.00594490	0.0000048841	7.23610
9	-0.139590	-0.000119340	0.00571285	0.0000048841	6.68223
10	-0.120272	-0.000117351	0.00500565	0.0000048841	5.13023
11	-0.101069	-0.000105859	0.00466310	0.0000048841	4.45210
12	-0.0841857	-0.000095362	0.00431171	0.0000048841	3.80640
13	-0.0642555	-0.000086853	0.00361335	0.0000048841	2.67323
14	-0.0509792	-0.000079118	0.00314704	0.0000048841	2.02778
15	-0.0337108	0	0	0	1.40897
16	-0.0198594	0	0	0	1.00601
17	-0.0143352	+0.000040001	-0.00175032	0.0000048841	0.627264
18	-0.0127847	0.000038233	-0.00163319	0.0000048841	0.546121
19	-0.0071280	0.000029835	-0.00116638	0.0000048841	0.273784
20	-0.0039483	0.000023641	-0.00081549	0.0000048841	0.136161
Total	-1.3913080	-0.0010717140	+0.05117920	0.0000732615	66.024260

TABLE VI E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR DETERMINATION OF $C_{L\alpha_1}$

Run 5301

Point Number	τq (dy)	$\frac{C_L}{2}$ y(dy)	$\frac{C_{n_t}}{2}$ dy	$(dy)^2$
4	0.120614	0.0142916	0.00246957	3.52688
5	0.155318	0.0350460	0.00211869	7.04903
6	0.201780	0.0503388	0.000742736	7.04903
7	0.221693	0.0658440	-0.000529673	7.04903
8	0.233375	0.0812430	-0.00131423	7.04903
9	0.223614	0.0888060	-0.00197292	6.05160
10	0.212933	0.0921816	-0.00233244	4.98182
11	0.198909	0.0964313	-0.00260625	4.34723
12	0.176299	0.0938784	-0.00261492	3.41510
13	0.156456	0.0895440	-0.00257480	2.68960
14	0.129267	0.0735900	-0.00234754	1.83603
15	0.108262	0.0721746	-0.00226588	1.41848
16	0.0822681	0.0610470	-0.00198161	0.938961
17	0.0695331	0.0529893	-0.00175061	0.670761
18	0.0569679	0.0444873	-0.00147620	0.450241
19	0.0485135	0.0392175	-0.00131306	0.337561
20	0.0365720	0.0307740	-0.00100841	0.198916
Total	2.4223750	1.0363840	-0.02075800	59.059300

TABLE VII E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR "TAIL OFF" CONSIDERATIONS

Run 5301

Point Number	$y\int yd\ t/\tau$	$w\int yd\ t/\tau$	$(\int yd\ t/\tau)\cdot(\int C_{n_t}d\ t/\tau)$	$(\int yd\ t/\tau)^2$
4	0.000560077	0.00885077	0.00000139216	0.0000236780
5	0.00201119	0.0263644	0.00000357544	0.000101324
6	0.00520651	0.0478822	0.00000704739	0.000328552
7	0.0109806	0.0778039	0.0000114692	0.000852990
8	0.0200673	0.115914	0.0000159849	0.00187454
9	0.0330517	0.150723	0.0000196617	0.00364647
10	0.0501049	0.182653	0.0000211185	0.00642338
11	0.0717637	0.219186	0.0000191117	0.0104788
12	0.0977098	0.242716	0.0000123288	0.0161214
13	0.127003	0.262953	−0.000000783972	0.0236298
14	0.159934	0.261085	−0.0000213926	0.0332041
15	0.194775	0.271112	−0.0000510325	0.0451010
16	0.23254	0.258608	−0.0000907200	0.0594092
17	0.270605	0.252760	−0.000141297	0.0763085
18	0.311095	0.238361	−0.000203341	0.0958769
19	0.352711	0.234671	−0.000277101	0.118226
20	0.395982	0.207343	−0.000368757	0.143368
Total	2.336101	3.058986	−0.001043	0.634975

TABLE VIII E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR "TAIL OFF" CONSIDERATIONS

Run 5301

Point Number	yw	$y\int C_{n_t}\, d\, t/\tau$	y^2
4	0.209355	0.0000329301	0.0132480
5	0.523306	0.0000709690	0.039920
6	0.758782	0.000111679	0.0825068
7	1.001573	0.00014764 34	0.141353
8	1.24088	0.000171121	0.214823
9	1.36616	0.000178214	0.299581
10	1.42476	0.000164732	0.290838
11	1.50109	0.000130886	0.491471
12	1.47107	0.0000747233	0.592207
13	1.41330	-0.00000421362	0.682606
14	1.25757	-0.000103042	0.770357
15	1.17083	-0.000220391	0.841164
16	1.01225	-0.000355097	0.910211
17	0.896334	-0.000501065	0.959616
18	0.773418	-0.000659786	1.00942
19	0.700109	-0.000826692	1.05227
20	0.57268	-0.0010185	1.09370
Total	17.293467	-0.002601	9.585292

TABLE IX E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR "TAIL OFF" CONSIDERATIONS

Run 5301

Point Number	$w \int C_{n_t} \, d\, t/\tau$	$(\int C_{n_t} \, d\, t/\tau)^2$
4	0.000520387	0.0000000818532
5	0.000930322	0.000000126167
6	0.00102707	0.000000151165
7	0.00104614	0.000000154213
8	0.000988441	0.000000136309
9	0.000812698	0.000000106015
10	0.000600517	0.0000000694432
11	0.000399762	0.0000000348569
12	0.000185616	0.00000000094284
13	-0.0000087240б	0.00000000002601
14	-0.000168211	0.000000013783
15	-0.000306767	0.000000057744
16	-0.000394904	0.000000138533
17	-0.000467702	0.000000261274
18	-0.000505528	0.000000431255
19	-0.000550027	0.000000649475
20	-0.000533308	0.000000948481
Total	0.0035766	0.00000339Ꮞ1

TABLE X E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR DETERMINATION OF LUMPED CONSTANTS

Run 5301

Point Number	$n_z \cdot \int n_z d\ t/\tau$	$(\int n_z d\ t/\tau)\cdot(\int \delta d\ t/\tau)$	$\delta \int n_z d\ t/\tau$
2	0.0000100890[illegible]	−0.000000253116	−0.0000770384
4	0.000136517	+0.0000001509[illegible]	+0.000011483692
6	0.00407180	0.0000253085	0.000149491
8	0.0187496	0.0000979277	0.000422363
10	0.0494262	0.000244047	0.000823513
12	0.098604	0.000486670	0.00133615
14	0.164264	0.000839491	0.00200424
16	0.241431	0.00131379	0.00272315
18	0.324695	0.00190813	0.00340961
20	0.414297	0.00262388	0.00418133
Total	1.316042	0.00754050	0.0150570

TABLE XI E

PRODUCTS AND SUMS OF BASIC VARIABLES FOR DETERMINATION OF LUMPED CONSTANTS

Run 5301

Point Number	$hdn_z \int n_z d\ t/\tau$	$(n_z d\ t/\tau)^2$	$n_z \int \delta\ d\ t/\tau$
2	-0.0000100036	0.00000005069山	-0.00000050374山
4	+0.0000451248	0.00000203918	+ 0.000101049
6	0.000545305	0.00020701山5	0.000497797
8	0.00142513	0.00158468	0.00115866
10	0.00232095	0.00594564	0.00202877
12	0.00324655	0.0156520	0.00307699
14	0.00344155	0.03300178	0.00417648
16	0.00326974	0.0600779	0.0052796
18	0.00307177	0.0978488	0.00633180
20	0.00187968	0.147155	0.00738720
Total	0.0192358	0.361491	0.03003331

TABLE XII E

P[illegible]DUCTS AND SUMS OF BASIC VARIABLES F[illegible]R DETERMINATION OF LUMPED CONSTANTS

Run 5301

Point Number	n_z (δ)	$n_z h d n_z$	n_z^2
2	−0.000[illegible]53319	−0.000199089	0.000200789
4	+0.000[illegible]3284	+ 0.00302096	0.00913936
6	0.00[illegible]4037	0.0107257	0.0800890
8	0.00[illegible]9731	0.0168618	0.221841
10	0.006[illegible]4588	0.0192941	0.410881
12	0.00[illegible]4788	0.02052645	0.625681
14	0.00[illegible]7112	0.0171218	0.817216
16	0.01[illegible]434	0.0131399	0.970225
18	0.01[illegible]442	0.0101932	1.077440
20	0.01[illegible]20	0.0052920	1.16640
Total	0.068[illegible]21	0.115977	5.379113

TABLE XXIII E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR DETERMINATION OF LUMPED CONSTANTS

Run 5301

Point Number	$\delta\int\delta\, d\ t/\tau$	$hdn_z\int\delta\, d\ t/\tau$	$(\int\delta\, d\ t/\tau)^2$
2	0.00000384651	0.000004499478	0.000000126380
4	0.0000109822	0.0000334012	0.00000111725
6	0.0000182760	0.0000666661	0.00000309408
8	0.0000261006	0.0000880680	0.00000605160
10	0.0000338022	0.0000952665	0.00000017225
12	0.0000415452	0.0001009455	0.0000151321
14	0.0000509586	0.0000875028	0.0000213444
16	0.0000595496	0.0000715024	0.0000287296
18	0.000066490	0.0000599020	0.000037210
20	0.0000745560	0.0000335160	0.0000467856
Total	0.000386107	0.000641765	0.000169608

TABLE XIV E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR DETERMINATION OF LUMPED CONSTANTS

Run 5301

Point Number	$\delta\,(hdn_z)$	δ^2
2	0.000152021	0.000117072
4	0.000328324	0.000107952
6	0.000393781	0.000107952
8	0.000379838	0.000112572
10	0.000321468	0.000114062
12	0.000277146	0.000114062
14	0.000208908	0.000121661
16	0.000148207	0.0001234321
18	0.000107028	0.000118810
20	0.000053410	0.000118810
Total	0.002370141	0.001156385

TABLE XV E

BASIC VARIABLES

Run 5068

Point Number	Time (sec.)	$n_z(g)$	dn_z	τq(rad.)	v*	$d\delta$
2	0.10	-0.01414	1.340	-0.0262	-0.0271	-0.02485
3	0.15	+0.0319	1.132	-0.0334	-0.0314	-0.02020
4	0.20	0.0921	1.953	-0.0494	-0.0435	0
5	0.25	0.1734	1.953	-0.0625	-0.0514	0.01728
6	0.30	0.2463	2.160	-0.0639	-0.0482	-0.00433
7	0.35	0.322	2.160	-0.0639	-0.0435	0
8	0.40	0.396	2.160	-0.0639	-0.0386	-0.26200
9	0.45	0.478	0.519	-0.0523	-0.0220	+0.02045
10	0.50	0.497	0.	-0.0392	-0.0074	+0.02800
11	0.55	0.494	-0.308	-0.0233	0.0083	0
12	0.60	0.464	-0.772	-0.0138	0.0159	0.00431
13	0.65	0.428	-1.234	-0.00872	0.0186	0
14	0.70	0.390	-1.234	0.00364	0.0285	0
15	0.75	0.350	-1.234	0.00654	0.0289	0
16	0.80	0.298	-1.234	0.00726	0.0263	0
17	0.85	0.248	-1.234	0.00872	0.0246	0
18	0.90	0.209	-1.338	0.00872	0.0221	0
19	0.95	0.159	-1.338	0.00872	0.0189	0
20	1.00	0.122	-0.722	0.00872	0.0165	0

* $v = \frac{C_L}{2} n_z + \tau q$

TABLE XVI E

BASIC VARIABLES

Run 5068

Point Number	Time (sec.)	y^*	$\frac{C_L}{2} \cdot y$	$C_{n_t}/2$	dy
2	0.10	-0.00397	-0.00025	-0.001158	-1.148
3	0.15	-0.04357	-0.00278	-0.000746	-1.816
4	0.20	-0.009938	-0.00634	+0.000465	-2.090
5	0.25	-0.17316	-0.01108	0.00001545	-2.090
6	0.30	-0.24547	-0.01568	0.00005305	-2.090
7	0.35	-0.31579	-0.02015	0.000398	-2.090
8	0.40	-0.37731	-0.02410	0.001193	-2.090
9	0.45	-0.45205	-0.02890	0.001660	-1.600
10	0.50	-0.47100	-0.03010	0.1662	-0.262
11	0.55	-0.47180	-0.03015	0.001420	+0.262
12	0.60	-0.44420	-0.02835	0.001265	0.386
13	0.65	-0.41184	-0.02630	0.001032	1.091
14	0.70	-0.37564	-0.02400	0.000886	1.211
15	0.75	-0.33880	-0.02165	0.000716	1.211
16	0.80	-0.28902	-0.01846	0.000542	1.211
17	0.85	-0.24236	-0.01549	0.000361	1.211
18	0.90	-0.20550	-0.01311	0.000224	1.211
19	0.95	-0.15679	-0.01002	0.000161	1.211
20	1.00	-0.12016	-0.00770	0.000104	1.211

$$^* \quad y = \frac{1}{C_L} C_{n_t} - n_z$$

TABLE XVII E

BASIC VARIABLES

Run 5068

Point Number	Time (sec.)	$\int y d\, t/\tau$	w^*	$\int C_{n_t}\, d\, t/\tau$	$\int q\, dt$
2	0.10	-0.0002065	-1.0924	-0.00007672	-0.00126
3	0.15	-0.0009735	-1.7802	-0.00014312	-0.00224
4	0.20	-0.0033435	-2.1124	-0.00018402	-0.00372
5	0.25	-0.0081235	-2.0907	-0.00019982	-0.00565
6	0.30	-0.0152535	-2.0928	-0.00019707	-0.00793
7	0.35	-0.0248035	-2.1091	-0.00018331	-0.01023
8	0.40	-0.0367235	-2.1473	-0.00013171	-0.01236
9	0.45	-0.0511235	-1.6799	-0.00003191	-0.01428
10	0.50	-0.0670235	-0.3415	+0.00008609	-0.01590
11	0.55	-0.0832235	+0.1933	0.00019209	-0.01696
12	0.60	-0.0991235	0.8253	0.00028219	-0.01752
13	0.65	-0.1138735	1.0414	0.00036029	-0.01788
14	0.70	-0.1273735	1.1684	0.00042629	-0.01799
15	0.75	-0.1394035	1.1766	0.00048069	-0.01781
16	0.80	-0.1500035	1.1850	0.00052269	-0.01758
17	0.85	-0.1591735	1.1937	0.00055399	-0.01725
18	0.90	-0.1669235	1.2002	0.00057464	-0.01697
19	0.95	-0.1731835	1.2033	0.00058839	-0.01665
20	1.00	-0.1780435	1.2060	0.00059734	-0.01639

* $w = dy - (C_{L\alpha_1} / 2C_L)\, C_{n_t}$

TABLE XVIII E

BASIC VARIABLES

Run 5068

Point Number	Time (sec.)	δ (rad.)	$\int \delta \, d\, t/\tau$	hdn_z	$\int n_z d\, t/\tau$
2	0.10	0.00926	0.000344	0.01915	-0.000757
4	0.20	0.00790	0.000931	0.0274	+0.001303
6	0.30	0.00837	0.001492	0.0308	0.013343
8	0.40	0.00185	0.001972	0.0288	0.035543
10	0.50	0.00028	0.001957	0.0021	0.067793
11	0.55	0.00028	0.001966	-0.0041	0.084693
12	0.60	0.00028	0.001966	-0.0113	0.101443
13	0.65	0	0.001966	-0.0171	0.116743
14	0.70	0	0.001966	-0.0171	0.130743
15	0.75	0	0.001966	-0.0171	0.143253
16	0.80	0	0.001966	-0.0253	0.153993
17	0.85	0	0.001966	-0.0171	0.163453
18	0.90	0	0.001966	-0.0178	0.171393
19	0.95	0	0.001966	-0.0151	0.177863
20	1.00	0	0.001966	-0.0113	0.182603

TABLE XIX E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR DETERMINATION OF C_{L_α}

Run 5068

Point Number	vdn_z	$vd\delta$	$(d\delta)(dn_z)$	$(d\delta)^2$	$(dn_z)^2$
2	−0.036314	0.000673435	−0.033299	0.0006160	1.795600
3	−0.0355448	0.000634280	−0.0228664	0.0004080	1.281424
4	−0.0849555	0	0	0	3.814209
5	−0.1003842	−0.000888192	+0.03374784	0.0002982	3.814209
6	−0.1041120	+0.000208706	−0.00935280	0.0000188	4.665600
7	−0.0939600	0	0	0	4.665600
8	−0.0833760	0.01011320	−0.5659200	0.0686000	4.665600
9	−0.0114200	−0.000451000	+0.01061355	0.0004185	0.269361
11	−0.0025564	0	0	0	0.094864
12	−0.0122748	+0.000068529	−0.00332732	0.00001855	0.595984
13	−0.0229524	0		0	1.522756
14	−0.0351690	0		0	1.522756
15	−0.0356626	0		0	1.522756
16	−0.0324542	0		0	1.522756
17	−0.0303564	0		0	1.522756
18	−0.0295698	0		0	1.790244
19	−0.0252882	0		0	1.790244
20	−0.0119130	0		0	0.521284
Total	−0.7882638	0.010359183	−0.59040413	0.07037805	37.378003

TABLE XXI E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR DETERMINATION OF $C_{L\alpha_1}$

Run 5068

Point Number	$\tau q(dy)$	$\frac{Cn_t}{2}\,dy$	$\frac{C_L}{2}\,y(dy)$	$(dy)^2$
2	0.0300776	0.00132938	0.00029044	1.31790
3	0.0606544	0.00135474	0.00504848	3.29786
4	0.103246	-0.000971850	0.0132506	4.36810
5	0.130625	-0.0000322905	0.0231572	4.36810
6	0.133551	-0.000110874	0.0327712	4.36810
7	0.133551	-0.000831820	0.0421135	4.36810
8	0.133551	-0.00249337	0.0503690	4.36810
9	0.0836800	-0.00265600	0.0462400	2.5600
11	-0.00607988	+0.000371330	-0.00788423	0.068382
12	-0.0122268	0.00112079	-0.0251181	0.784996
13	-0.00951352	0.00112591	-0.0286933	1.190280
14	+0.00440199	0.00107295	-0.0290640	1.46652
15	0.00791994	0.000867076	-0.0262182	1.46652
16	0.00879186	0.000656362	-0.0223551	1.46652
17	0.0105599	0.000437171	-0.0187584	1.46652
18	0.0105599	0.000293062	-0.0158762	1.46652
19	0.0105599	0.000194729	-0.0121342	1.46652
20	0.0105599	0.000126307	-0.00932470	1.46652
Total	0.844469	0.00185360	0.0178140	41.325558

TABLE XXI E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR "TAIL OFF" CONSIDERATIONS

Run 5068

Point Number	$y\int yd\ t/\tau$	$w\int yd\ t/\tau$	$(\int yd\ t/\tau)\cdot(\int Cn_t d\ t/\tau)$
2	0.000000819805	0.000225581	0.00000000158427
3	0.0000424154	0.00173302	0.000000139327
4	0.000332260	0.00706264	0.000000615271
5	0.00140665	0.0169841	0.0000162324
6	0.00374426	0.0319224	0.00000300601
7	0.00783270	0.0523131	0.00000454673
8	0.0138561	0.0788564	0.00000483685
9	0.0231104	0.0858824	0.00000163135
11	0.0392648	-0.0160871	-0.0000159864
12	0.0440307	-0.0818017	-0.0000279717
13	0.0468977	-0.118588	-0.0000410275
14	0.0478466	-0.148823	-0.0000542980
15	0.0472299	-0.164022	-0.0000670099
16	0.0433540	-0.177754	-0.0000784053
17	0.0385773	-0.180002	-0.0000881805
18	0.0343019	-0.200346	-0.0000959209
19	0.0271526	-0.208388	-0.000101899
20	0.0213937	-0.214720	-0.000106353
Total	0.440375	-1.235552	-0.000660637

TABLE XXII E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR "TAIL OFF" CONSIDERATIONS

Run 5068

Point Number	$(\int y d\ t/\tau)^2$	yw	$y\int C_{n_t}\ d\ t/\tau$
2	0.0000000426423	0.00433683	0.000000304578
3	0.000000947702	0.0775633	0.0000062357 4
4	0.0000111790	0.209915	0.0000182870
5	0.0000659913	0.362029	0.0000346004
6	0.000232669	0.513715	0.0000483746
7	0.000615214	0.666033	0.0000578875
8	0.00134862	0.810198	0.0000496955
9	0.00261361	0.759399	0.0000144249
11	0.00692615	−0.0911989	−0.0000906281
12	0.00982547	−0.366576	−0.000125349
13	0.0129672	−0.428390	−0.000148382
14	0.0162240	−0.438898	−0.000160132
15	0.0194333	−0.398632	−0.000162858
16	0.0225105	−0.342489	−0.000151068
17	0.0253362	−0.289300	−0.000134265
18	0.0278635	−0.246641	−0.000118086
19	0.0299925	−0.188656	−0.0000922507
20	0.0316995	−0.144913	−0.0000717764
Total	0.207657	+0.466995	−0.00102498

TABLE XXIII E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR "TAIL OFF" CONSIDERATIONS

Run 5068

Point Number	y^2	$w\int C_{n_t}\, d\, t/\tau$	$(\int C_{n_t}\, d\, t/\tau)^2$
2	0.0000157609	0.0000838089	0.00000000588596
3	0.00189834	0.000254782	0.0000000204833
4	0.00987539	0.000388715	0.0000000338634
5	0.0299837	0.000417772	0.0000000399280
6	0.0602550	0.000412426	0.0000000388366
7	0.0997233	0.000386619	0.0000000336026
8	0.142363	0.000282821	0.0000000173475
9	0.204349	0.0000536056	0.00000000101825
11	0.222595	0.0000371310	0.0000000368986
12	0.197314	0.000232877	0.0000000796312
13	0.169612	0.000375206	0.000000129809
14	0.141105	0.000498077	0.000000181723
15	0.114785	0.000565580	0.000000231063
16	0.0835326	0.000619383	0.000000273205
17	0.0587384	0.000661287	0.000000306905
18	0.0422282	0.000689700	0.000000330211
19	0.0245815	0.000707998	0.000000346203
20	0.0114384	0.000720392	0.000000356815
Total	1.617394	0.007389376	0.00000246343

TABLE XXIV E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR DETERMINATION OF LUMPED CONSTANTS

Run 5068

($d\delta$ and δ assumed to = 0)

Point Number	$n_z \int n_z d\ t/\tau$	$(\int n_z d\ t/\tau)\cdot(\int \delta\ d\ t/\tau)$	$h d n_z \int n_z d\ t/\tau$
2	0.000010704	-0.0000000260408	-0.0000134897
4	0.000120006	+0.00000121309	+0.0000338780
6	0.00328638	0.0000199078	0.000383611
8	0.0140750	0.0000700908	0.00102186
10	0.0336931	0.000132671	0
12	0.0470696	0.000199437	-0.00104081
14	0.0509244	0.000257041	-0.00214549
16	0.04 58129	0.000302750	-0.00252703
18	0.0358211	0.000336959	-0.00305080
20	0.0222410	0.000358997	-0.00175299
Total	0.253054	0.00167891	-0.00909126

TABLE XXV E

PRODUCTS AND SUMS OF BASIC VARIABLES
FOR DETERMINATION OF LUMPED CONSTANTS

Run 5068

($d\delta$ and δ assumed to = 0)

Point Number	$(\int n_z d\, t/\tau)^2$	$n_z \int \delta d\, t/\tau$	$n_z h d n_z$
2	0.000000573049	-0.00000486416	-0.000251975
4	0.0000169781	+0.0000857451	+0.00239460
6	0.000178036	0.000367480	0.00708113
8	0.00126330	0.000780912	0.011385
10	0.00459589	0.000972629	0
12	0.0102907	0.000912224	-0.00476064
14	0.0170937	0.000765757	-0.00639170
16	0.0237138	0.000584885	-0.00488198
18	0.0293756	0.000410894	-0.00372020
20	0.0333439	0.000239459	-0.00116928
Total	0.119856	0.00511512	-0.000315045

TABLE XXVI E

PRODUCTS AND SUMS OF BASIC VARIABLES FOR DETERMINATION OF LUMPED CONSTANTS

Run 5068

($d\delta$ and δ assumed to = 0)

Point Number	n_z^2	$hdn_z \int \delta \, d \, t/\tau$	$(\int \delta \, d \, t/\tau)^2$
2	0.000199940	0.00000613008	0.000000118336
4	0.00848241	0.0000242060	0.000000866761
6	0.0606637	0.0000428950	0.00000222606
8	0.156816	0.0000566950	0.00000388878
10	0.247009	0	0.00000382985
12	0.215296	-0.0000201712	0.00000386516
14	0.151710	-0.0000322621	0.00000386516
16	0.0885063	-0.0000322621	0.00000386516
18	0.043681	-0.0000349948	0.00000386516
20	0.0148352	-0.0000188736	0.00000386516
Total	0.987200	-0.0000188736	0.00000386516